4.3.3 Taper Ratio

Tactical fighters of the Second World War required an adequate reserve of (thrust – drag) in a high-g turn. In this flight condition induced drag is very high, as will be discussed later in Chapter 17. As Raymer states, the spanwise load distribution for minimum induced drag is elliptic. Proof of this can be found in Ref. 4.3.3.1, Appendix C.5. To achieve this spanwise distribution at all lift coefficients, an untwisted, unswept wing with an elliptic area distribution is required. R.J. Mitchell, the chief designer at Supermarine Aviation, believed that it was worth paying the extra manufacturing cost of a curved planform in order to achieve this. The result was the beautiful wing planform of the Supermarine Spitfire, illustrated in Fig 4.3.3.1.

source: www.airmuseumuk.org

Fig 4.3.3.1 Supermarine Spitfire

A useful way of estimating spanwise lift distribution is through the use of Schrenk's Rule (described in Raymer Section 14.3.3, where he describes it as an approximation, which it is), using the following steps

- Plot the wing chord as function of semispan
- Add a quarter-ellipse with the same area
- Draw a line midway between the wing chord and ellipse
- This line represents the spanwise lift distribution

For an untapered wing ($\lambda = 1.0$) the lift distribution will look something like the dashed line in Fig. 4.3.3.2. The curve showing the lift distribution is the same shape as that for the corresponding curve in Raymer Fig. 4.23.

Raymer's statement that an untapered wing "loads up" the tip refers to the spanwise distribution of lift, not to the lift per unit area. On a small wing section of width δb and chord c, the lift is δL, and section lift is lift per unit span, or:

$$\text{Section lift} = \frac{\delta L}{\delta b}$$

The section lift coefficient is defined in Raymer Eq. (4.1) as

$$C_l = \frac{\text{Section lift}}{qc} = \frac{\delta L}{q\delta bc}$$

So we can see that where the ratio (Section lift)/c is high, i.e. near the wing root, the C_l is high, and where the ratio (Section lift)/c is low (near the wing tip), the C_l is low. This results from the

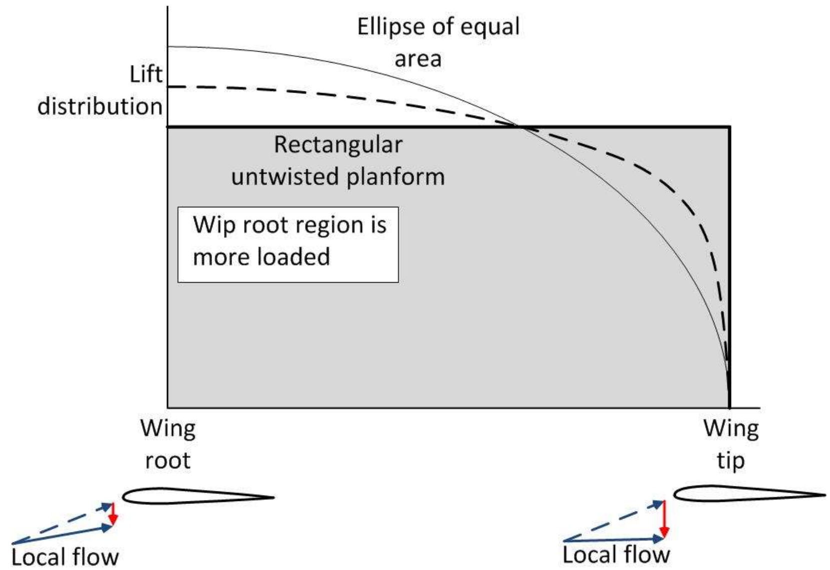

Fig. 4.3.3.2 Schrenk's Rule Applied to Untapered Wing

increased downwash near the wing tip due to the strong tip vortex (as illustrated at the bottom of Fig. 4.3.3.2). This is acceptable for a light airplane, because it ensures that the wing root stalls before the wing tip, so that the stall will be symmetrical, and benign.

For a delta wing for which $\lambda = 0$ (Fig. 4.3.3.3), the opposite is true, and the wing tip has a higher C_l than the wing root. Again compare the curve for lift distribution with the curve in Raymer Fig. 4.23.

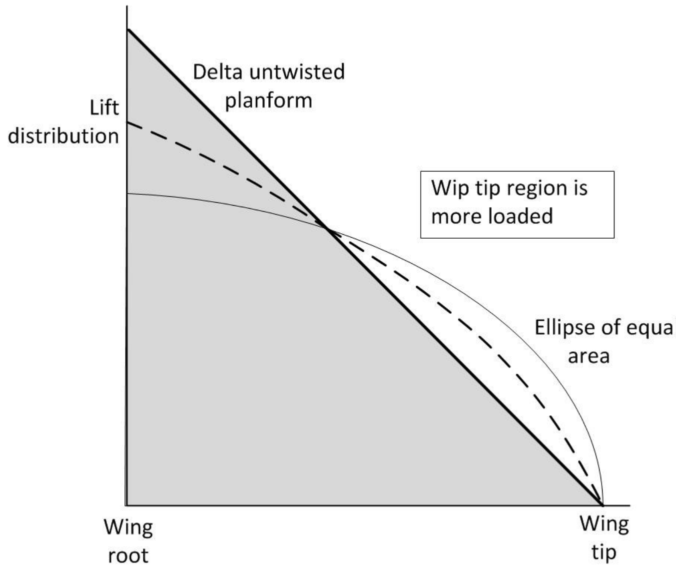

Fig. 4.3.3.3 Schrenk's Rule Applied to a Delta Wing

This can result in both poor handling qualities near the stall, and an increase in high subsonic drag. This was the case for the Avro Vulcan, which required significant modification to the wing planform to unload the wing tip to reduce high subsonic drag, as illustrated in Fig. 4.3.3.4.

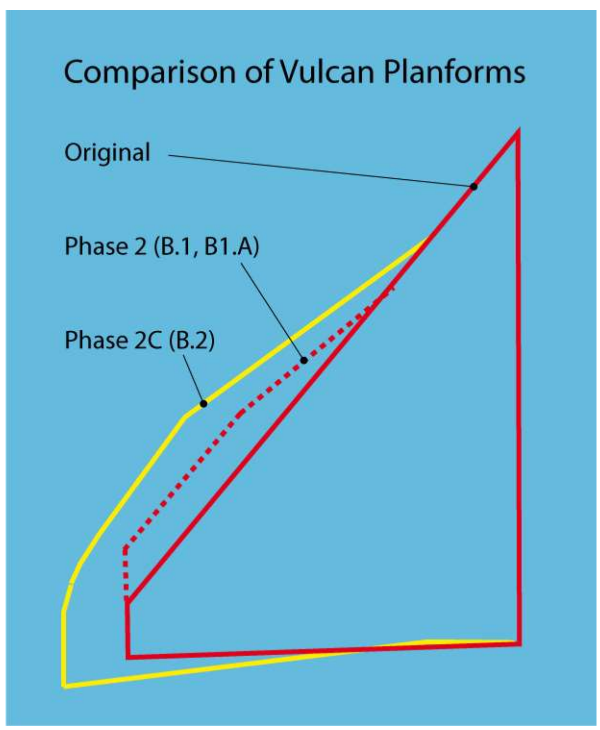

Fig. 4.3.3.4 Comparison of Vulcan Planforms

This modified planform can be seen in the sketch of Raymer Fig. 3.6.

References

4.3.3.1 Sforza, P.M., “Commercial Airplane Design Principles”. Elsevier, 2014.