16.4.2 Lateral-Trim Analysis

For the first design iteration, the vertical tail may be sized by assuming the same vertical tail volume coefficient as aircraft in the same class, as described in Raymer, page 121. The definition of vertical tail area is shown in Raymer Fig. 6.2, in which the height of the vertical tail extends from the intersection of the upper fuselage surface and the mid-chord of the vertical tail to the top of the tail. Sometimes (e.g. in Raymer Eq. (16.51), or Torenbeek Fig. 9.20) the root of the vertical tail is assumed to be at the aircraft centerline, and heights and areas are calculated accordingly. When evaluating data, make sure the definition of the vertical tail area is understood.

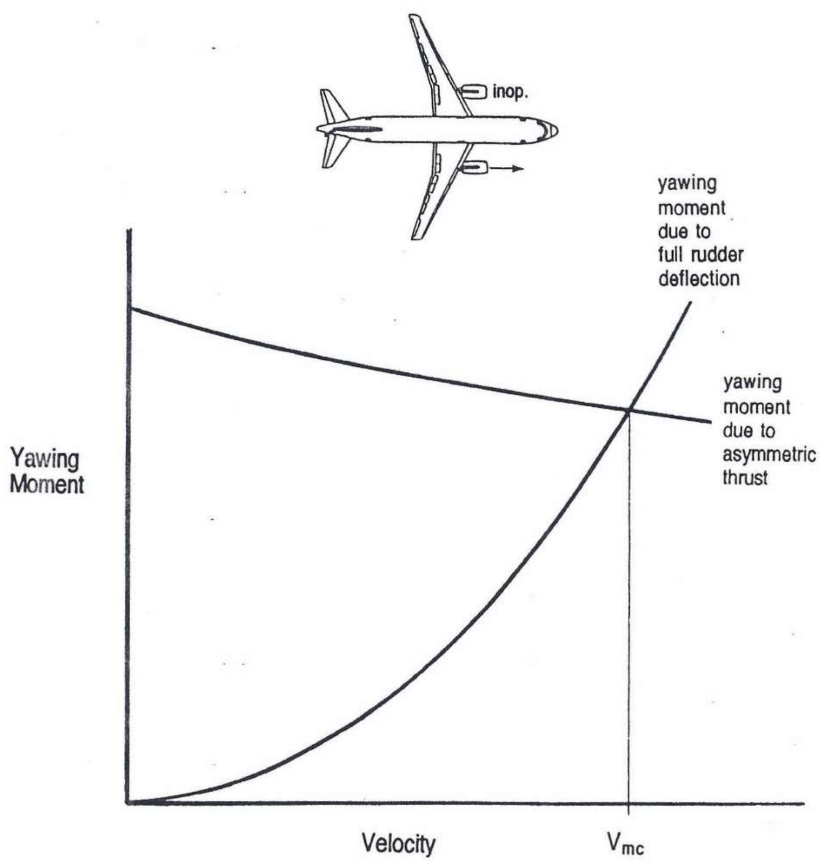
The next level of analysis will depend on the type of aircraft. For single engine aircraft, or one with multiple engines mounted close to the centerline, the vertical stabilizer may be sized by the desired value of C_{n_β}, using goal values suggested in Fig. 16.20. For multi-engine aircraft with engines mounted on the wing, the requirement for one-engine inoperative (OEI) climb after takeoff may size the tail.

The first method is as follows:

- Select a goal value for C_{n_β} from Fig. 16.20.
- Using Eq. (16.42), calculate $C_{n_{\beta_v}}$. In this equation, the first term on the right hand side ($C_{n_{\beta_w}}$) may be found using Eq. (16.44), using values for a typical cruise condition. The second term ($C_{n_{\beta_{fuse}}}$) may be found using Eq. (16.50). The final term, which is the yawing moment coefficient due to the inlet side-force, is a bit more difficult. Inlet side-force (F_{p_β}) may be found using Eq. (16.28), assuming that $F_{p_\beta} = F_{p_\alpha}$. The change in local airflow angle at the inlet as a function of yaw angle ($\frac{\partial \beta_p}{\partial \beta}$) depends on the type of inlet. For a pitot inlet on the nose of the airplane, assume the value is unity. For inlets on the side of the fuselage, assume a value of zero. Wing-mounted engines will require wind-tunnel analysis or CFD to calculate the correct value. As an estimate, a value of 0.5 is suggested. Use an aftmost value of $\overline{X}_{cg}$.
- With a value of $C_{n_{\beta_v}}$ now determined, Eq. (16.39) may be used to find S_v. Unfortunately S_v appears twice in the equation, the first time embedded in the sideslip derivative, and then directly, so an iterative method can be used to find a solution. Assume that the lateral lift force derivative C_{F_β} is equivalent to C_{L_α} in longitudinal notation, so use Eq. (12.6) with the appropriate geometric data for the vertical stabilizer. For the sideslip derivative ($\frac{\partial \beta_v}{\partial \beta}\eta_v$) use Eq. (16.51), which itself contains a modified tail area. In Eq. (16.51) $\frac{Z_{wf}}{D_f}$ is the height of the wing from the fuselage centerline divided by the fuselage diameter. In Eq. (16.30)

$\overline{X}_{acv}$ is the non-dimensional longitudinal location of the aerodynamic center of the vertical tail.

The second method is applicable for multi-engine airplanes, and will probably set the vertical stabilizer sizing requirement. Loss of power on the number one engine (engines are numbered from port to starboard) requires that the pilot simultaneously apply right rudder to correct the yawing moment due to asymmetric thrust (see Fig. 16.4.2.1), and apply a rolling moment to the right both to hold the starboard wing low (to prevent slideslip) and balance the rolling moment due to the rudder. The low starboard wing produces a side force on the airplane which balances the side force generated by the rudder deflection. If the thrust is lost soon after takeoff, the remaining engine (or engines) must produce enough thrust to maintain the required rate of climb with the additional drag due to rudder and aileron deflection. This procedure is a simplification of that described in Raymer's Chapter 16 Lateral-Trim Analysis subsection. Eq. (16.38) neglects to include the yawing moment coefficient due to rudder deflection ($C_{n_{\delta_r}}$), which limits its usefulness for one-engine inoperative calculations.



Source: Schaufele

Fig. 16.4.2.1 Air Minimum Control Speed

The following conditions are specified in FAR 25.149(c) for minimum control speed:

- $V_{MC} \le 1.13\ V_{SR}$ (V_{SR} is reference stall speed, which is the stall speed in that particular flight condition, which in this case is with takeoff flaps) (see Fig.16.4.2.1)
- Maximum takeoff thrust on remaining engines
- Most unfavorable c.g. (usually aft cg, where the tail moment arm is the shortest)
- Aircraft trimmed for takeoff (takeoff flap setting)
- Maximum TOGW
- Most critical takeoff configuration existing along the flight path, except for landing gear retracted.

You can make certain simplifying assumptions:

- Drag increment and adverse yawing moment ($C_{n_{\delta_a}}$) due to aileron deflection (required to maintain low wing on the operating engine) will be neglected. For a more precise calculation these terms should be included.
- Sideslip (β) will be assumed to be zero.

Taking moments about the Z axis passing through the c.g. in Raymer Fig. 16.19

$$F_V\ l_V = \left(T + D_{engine\ out}\right)\ y_E \qquad (16.4.2.1)$$

where F_v = required rudder side force
l_V = distance from c.g. to MAC of vertical stabilizer
T = thrust of operating engine at V_{MC}
$D_{engine\ out}$ = windmill drag of dead engine
y_E = lateral distance from engine centerline to inoperative engine.

For a high-bypass ratio turbofan, takeoff windmill drag may be approximated as 1% of takeoff thrust (Ref 16.4.2.1., Section 2.3.2).

Rudder side force may be defined as:

$$F_V = \frac{\partial C_F}{\partial \delta_r}\,\delta_r\, q\, S_v \qquad (16.4.2.2)$$

where δ_r is the maximum rudder deflection, which may be in the range $25^0 – 30^0$ (Torenbeek, p. 335). Figure 16.4.2.1 states "full rudder deflection", but Raymer suggests 20^0 to be conservative and provide some margin for control.

The change in force per unit rudder deflection can be assumed to be same as that of a full-span flap, so Raymer's Eq. (16.17) may be modified to the form:

$$\frac{\partial C_F}{\partial \delta_r} = 0.9\, K_r \frac{\partial C_l}{\partial \delta_r} \cos\Lambda_{HL} \qquad (16.4.2.3)$$

where δ_r = rudder deflection in radians
$K_r = K_f$ (Fig. 16.7)
$\frac{\partial C_l}{\partial \delta_r} = \frac{\partial C_l}{\partial \delta_f}$ (Fig 16.6)
Λ_{HL} = Sweep of the rudder hinge line

The procedure for calculating tail area is

- Estimate $C_{L_{max}}$ with takeoff flaps (a range of values is given in the annotation to Section 17.3.1)
- Calculate $V_{S_{TO}}$. Then $V_{MC} = 1.13\ V_{S_{TO}}$ (which is the stall speed in the takeoff condition)
- Calculate the thrust per engine at that speed from engine data (such as Appendix E.2, scaled appropriately)
- Calculate the yawing moment due to engine thrust, plus drag from the dead engine, using Eq. (16.4.2.1) above
- Calculate $\frac{\partial C_F}{\partial \delta_r}$ from Eq. (16.4.2.3)
- Hence calculate S_V from Eq. (16.4.2.2)

If the tail area is less than the value first calculated from the tail volume coefficients of comparable aircraft, then it is possible that the tail volume for this class of aircraft is set by Dutch roll requirements, and the larger value should be used.

In the event of engine failure during takeoff, the aircraft must also meet the climb requirements of FAR 25.121(b). These are:

- 2.4% for a two-engine airplane
- 2.7% for a three-engine airplane
- 3.0% for a four-engine airplane

Trim drag due to rudder deflection and to a lesser extent aileron deflection may be critical in meeting these climb requirements. This issue is discussed in annotations to Section 17.3.1.

The second vertical tail sizing requirement, but which is harder to calculate, is the requirement to maintain directional control while on the ground. The tail must be sized such that the minimum control speed on the ground (V_{MCG}) is less than the takeoff decision speed (V_1). If this is not the case, and V_{MCG} is greater than V_1, then the situation may arise in which critical engine failure occurs above V_1. In this case the pilot has to continue the takeoff (because V_1 has been exceeded), but the airplane lacks adequate lateral control while still on the ground.

The requirements for V_{MCG} are given in FAR 25.149(e). They are to lose the most critical engine, apply the rudder (without the use of nosewheel steering), and maintain control down the runway with a maximum of 30 ft lateral deviation from the runway centerline. In flight test, this is performed at successively lower speeds until the pilot can no longer maintain 30 ft lateral deviation.

The third minimum control speed, which is specified in FAR 25.149(f), is the minimum control speed on approach to land (V_{MCL}). V_{MCL} is the calibrated airspeed at which, when the critical engine is suddenly made inoperative, it is possible to maintain control of the airplane with that engine still inoperative, and maintain straight flight with an angle of bank of not more than 5^0. V_{MCL} must be established with:

- The airplane in the most critical configuration
- The most unfavorable center of gravity
- Trimmed for approach with all engines operating
- Go-around thrust on the operating engine(s)

There are additional requirements if the airplane has more than two engines, as specified in FAR 25.149(g).

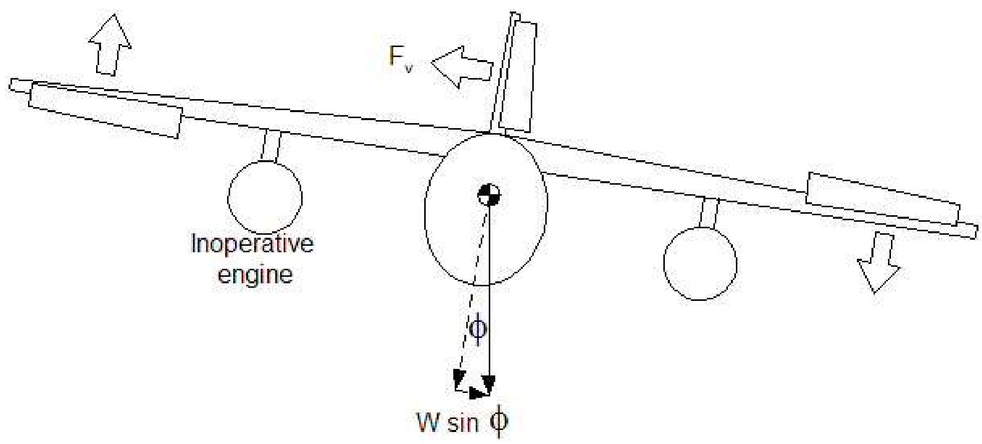


Fig. 16.4.2 Lateral forces in engine-out operation

The required bank angle, φ, may be calculated simply from consideration of forces parallel to the y-axis

$$F_v = W \sin \varphi \qquad (16.4.2.4)$$

The low thrust setting on approach reduces F_v and thus φ, but the low aircraft weight tends to increase the required value of φ. The problem is particularly acute if the pilot has to make a go-around. The pilot must apply judicious thrust increase to avoid wing strike. If the 5^0 bank angle limit is exceeded at maximum TOGW, the designer has few options except to move the engines closer to the airplane centerline, or increase the vertical tail moment arm, both of which will increase airplane weight.

If additional control power is required for a derivative design (such as having increased engine thrust), improved rudder effectiveness may be achieved by adding vortex generators to the vertical stabilizer (as was done on the L1011 for one customer with particularly short runway, and reduced V_{MC}, requirements). If that doesn't work, a double-hinged rudder might do the trick. This was done on the Boeing 747SP (along with increased tail height) to make up for the shortened fuselage (and thus reduced rudder moment arm) and on the DC-10.

Sizing the Ailerons

To size the ailerons, you must first decide what the requirement is. Select the class of airplane from Table 16.2. Neglect roll acceleration effects so that the required roll can be translated into a steady roll rate (*P*).

Roll rate is related to rolling moment coefficients from the steady roll rate equation (and remembering that C_l refers to rolling moment coefficient and not section lift coefficient):

$$P = \frac{C_{l_{\delta_\alpha}}}{C_{l_P}} \delta_\alpha \tag{16.64}$$

you can obtain the value of the roll damping parameter C_{l_P} from Fig. 16.26, and assume a maximum aileron deflection of 20^0 (Torenbeek, page 257). This now leaves you with the hardest part, which is to determine the rolling rate coefficient due to aileron deflection.

This is defined in Eq. 16.48 as

$$C_{l_{\delta_\alpha}} = \frac{2\,\Sigma K_f \left(\frac{\partial C_L}{\partial \delta_f}\right)' Y_i\, S_i \cos\Lambda_{H\cdot L\cdot}}{S_w\, b} \tag{16.48}$$

in which the correction terms and lift coefficient derivatives are assumed to be the same for ailerons as for flaps, and where the limits to the summation are the distances of the inboard and outboard ends of the ailerons from the aircraft roll axis, and where:

K_f = Empirical correction for plain flap lift increment (Fig. 16.7)
$\left(\frac{\partial C_L}{\partial \delta_f}\right)' = \left(\frac{\partial C_l}{\partial \delta_f}\right)'$ = Theoretical lift increment for plain flaps applied to that strip (Fig. 16.6)
Y_i = Lateral geometric location of chordwise strip of area S_i (see Fig 16.22)
S_i = area of strip
$\Lambda_{H\cdot L\cdot}$ = Sweep of hinge line

S_w = Reference wing area
b = Span

Note that in Fig. 16.6 C_l really is the section lift coefficient, and not a rolling moment coefficient.

Note also that c_f/c is usually constant across the span (and is usually no greater than about 30%), in which case K_f is constant. The value of t/c is also usually constant across the span (at least in the aileron area), so that the theoretical lift increment due to aileron deflection is also constant. In this case Eq. (16.48) can either be converted into an integral format and solved, or solved using a spreadsheet. The ailerons hinges are usually mounted to the rear spar and extend out to the tip fairing. This sets the maximum value of Y_i. The inboard value of Y_i is then selected that meets the roll rate requirement in Table 16.2. The ailerons should be no larger than necessary in order to leave the remainder of the trailing edge for flaps and inboard aileron.

References:

16.4.2.1 Daggett, D.L., et al., "Ultra-Efficient Engine Diameter Study", NASA/CR-2003-212309, May 2003