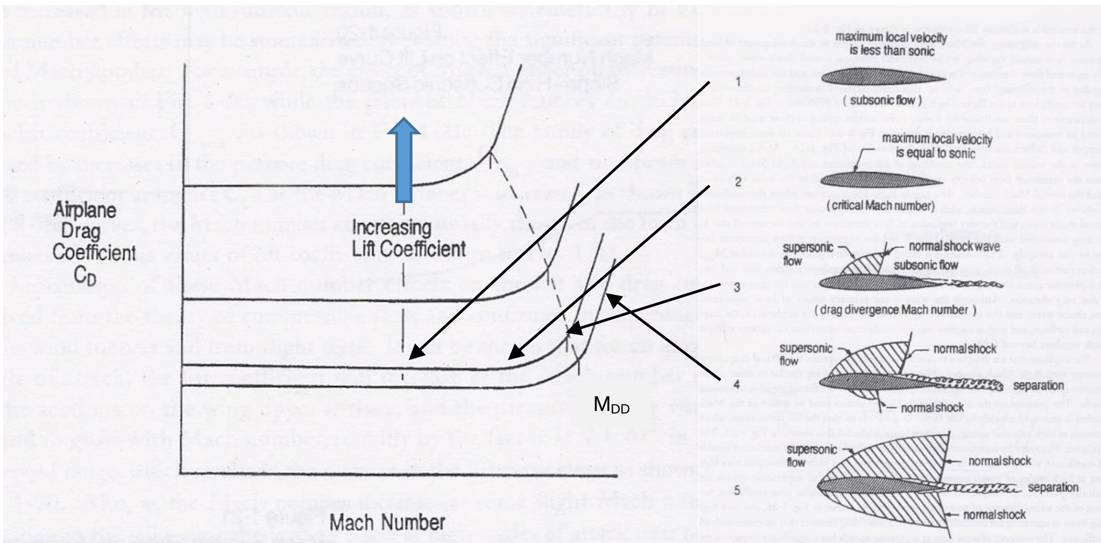
12.5.10 Generation of the Drag Map

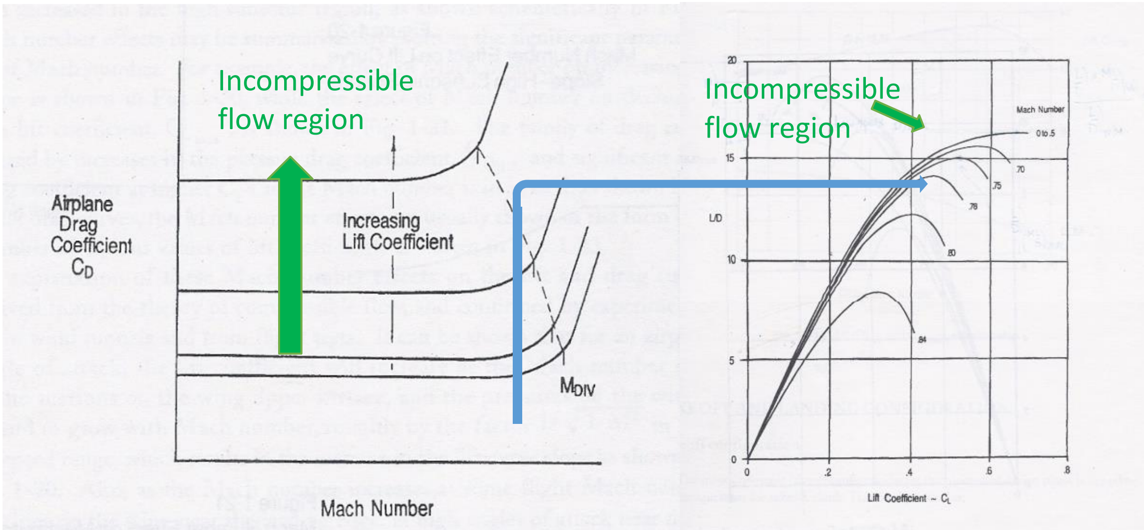
The measure of aerodynamic efficiency of an aircraft may be defined in the Breguet range equation as *ML/D*. To maximize range for a given structural weight fraction, the effect of speed on engine specific fuel consumption must also be included. An enormous amount of time and effort is expended in increasing *ML/D* by an amount as small as 0.01. Billions of dollars in aircraft sales (as exemplified by the ongoing battle between the 737 and A320) can depend on which airplane has the higher value.



Source: Schaufele

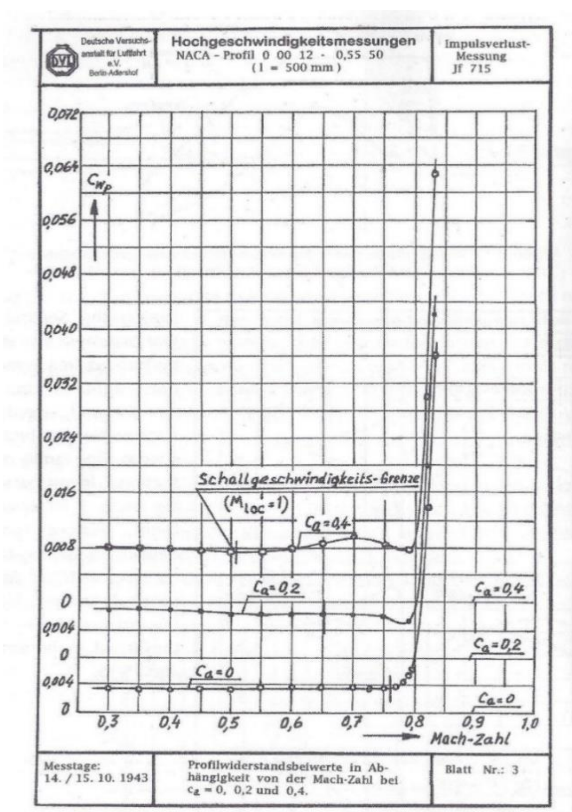
Figure 12.5.10.1 Typical Drag Map

A useful way to define the transonic characteristics of an aircraft is by means of the drag map (Fig. 12.5.10.1), which shows the airfoil or airplane drag coefficient and in particular the drag rise (increase in compressibility drag, C_{D_C}) with increasing Mach number for different lift coefficients, in the region of the drag divergence Mach number, M_{DD} (or M_{DIV} in Schaufele terminology). For a given wing thickness/chord ratio (*t/c*) and sweep (Λ) the aerodynamicists in the design team try to increase the Mach number at which drag rise occurs. From a knowledge of the drag map, three derivative plots can be generated: the drag polars as a function of Mach number, a plot of *L/D* as a function of C_L for different values of Mach number (a simple cross-plot of drag map data, as illustrated in Fig. 12.5.10.2), and a similar plot of *ML/D* as a function of C_L, so that the maximum value of *ML/D* can be determined. In addition the sensitivity of the design parameters to these plots can be investigated.



Source: Schaufele

Figure 12.5.10.2 Derivative Plot of L/D vs. C_L for Different Mach Values

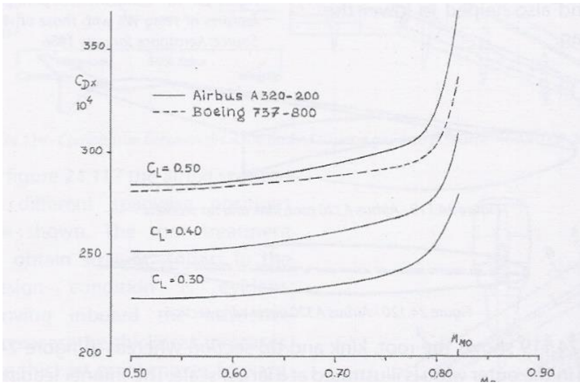


Source: Obert

Figure 12.5.10.3 Drag Map for NACA 0012 Airfoil

Presentation of data in this format appeared early in the investigation of airfoils in compressible flow, as illustrated in Fig. 12.5.10.3, dated 15 October 1943. This, and other drag maps (Fig. 12.5.10.4) appear in Obert (Ref. 4) in at least thirty instances, indicating the importance of this data format. This figure also illustrates that the shape of the drag rise curve can take many forms.

The drag map can be particularly useful in comparing the high-speed aerodynamic efficiency of similar aircraft, as illustrated in Fig. 12.5.10.4. Unfortunately, the lack of industry consensus on definitions adds to the difficulty of comparing values.



Source: Obert

Figure 12.5.10.4 Drag Rise Comparison Between B.737 and A320

The notation of Raymer (Ref. 12.5.10.1) is used. M_{crit} is the Mach number at which compressibility drag first appears (Fig. 12.5.10.1). $M_{DD_{Boeing}}$ is the Mach number at which compressibility drag reaches 20 counts (ΔC_{D_C} = 0.0020). $M_{DD_{Douglas}}$ is the Mach number where the gradient of the C_{D_C} vs. Mach curve has a gradient of 0.10. The shape of the A320 drag rise is fairly typical for commercial transports with supercritical wing sections, although there are differences between aircraft types. Examples of differences between these values are shown in Table 12.5.1.1, taken from Fig. 12.5.10.4 at C_L = 0.5. The values in the last two column are somewhat different from those in Raymer Section 12.5.10, suggestive of the variation of these values. The reality is that the differences in values can vary widely, depending on the configuration design. The Douglas definition has gained greater acceptance, and if a value is stated without its definition, it can generally be assumed to be based on the Douglas definition (i.e. M_{DD} occurs at dC_{D_C}/dM = 0.10), and for the rest of this paper, that will be the case.

Aircraft	$M_{DD_{Douglas}}$	$M_{DD_{Boeing}}$	M_{crit}	$M_{DD_{Douglas}} - M_{DD_{Boeing}}$	$M_{DD_{Boeing}} - M_{crit}$
A320-200	0.80	0.78	0.575	0.020	0.205
B.737-800	0.805	0.80	0.50	0.005	0.30

Table 12.5.10.1 Examples of Differences in Drag Divergence Mach Number values

12.5.10.1 Drag Map Generation

To generate a drag map using a spreadsheet, the relationship between C_{D_c} and (M-M_{DD}) must be expressed algebraically. Figures 12.5.10.3 and 12.5.10.4, and Ref. 10.5.10.6, Fig. 12.14, show that this relationship is a weak function of C_L and wing design. In particular it is a function of wing sweep, with a slightly larger knee radius for higher wing sweep. The simplifying assumption must be made that it is independent of C_L and wing design. An empirical equation which was derived by adjustment of the constants in the equation, and which may be used to plot a drag map is given by Eq. 12.5.10.1. This is broadly similar to the shape suggested by Schaufele, but adjusted to meet the Douglas definition of M_{DD}, (dC_{D_C}/dM = 0.10) which the Schaufele curve does not, and with a knee radius more appropriate for modern airfoil sections.

$$C_{D_c} = 0.04\left(\frac{(M - M_{DD}) + 0.308}{0.36}\right)^{22} + 0.017\big((M - M_{DD}) + 0.308\big)^{2.5} \quad (12.5.10.1)$$

Small changes in the shape of the drag rise can have a significant effect on the Mach number for maximum ML/D on a plot of ML/D as a function of C_L and M. This function is not valid when $(M-M_{DD}) < -0.3$, so the value of C_{D_C} must be set to zero for that condition in a spreadsheet using an Excel conditional function:

IF(condition, value_if_true, value_if_false)

It will also only be valid for Mach numbers up to about $M_{DD} + 0.04$.

Equation (12.5.10.1) is plotted in Fig. 12.5.10.5 and compared with the drag rise curve due to Schaufele (Ref. 12.5.10.3) and Lock (Ref. 12.5.10.5). The Power function and Schaufele curves are reasonable fits to the drag rise of commercial aircraft. Note that the value of M_{DD} on Schaufele's curve does not quite meet either the Boeing or Douglas definitions of M_{DD}. Schaufele's curve only exists in a tabular format, so the equation above is preferred for generating a drag map. Equation 12.5.10.1 is a better fit to drag rise curves for a configuration with a supercritical wing than Lock's 4^{th} Power function, used in Ref. 12.5.10.5 to generate a drag map. Lock's function matches the drag rise for a wing with a NACA 0012 airfoil and 35^0 sweep at low values of C_L (0 to 0.2), but is not a good match at higher lift coefficients. In addition, it does not exhibit drag creep (Fig.12.5.10.4) which is evident for many configurations, and underestimated by Schaufele. Both the Power Function and Lock's 4^{th} Power meet the Douglas definition of M_{DD}. A discussion of different procedures for determining C_{D_C} as a function of $(M-M_{DD})$ may be found in Sforza (Ref. 12.5.10.7), but none of these procedures generate curves that are representative of supercritical wing configurations.

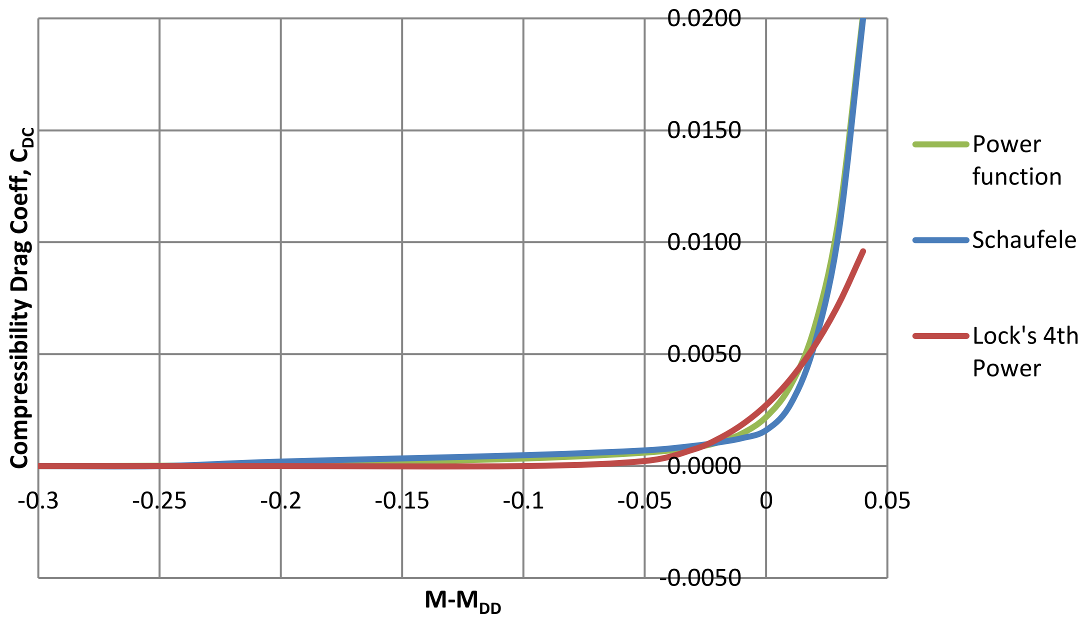


Figure 12.5.10.5 Drag rise comparison, C_{D_C} vs. $(M-M_{DD})$

In Ref. 12.5.10.1, Section 12.5.10, Raymer describes a method for determining the value of M_{DD} (Boeing definition) for a given wing design. This includes adjustments to M_{DD} for different values of C_L and t/c (Raymer Fig. 12.30). Provided that the shape of the drag rise is known (or assumed), it is therefore possible to generate a drag map based on the methods in this section. However, Raymer's method is not readily amenable to the use of MATLAB or a spreadsheet, so its use is not recommended.

An alternative is to use Schaufele Fig. 4-8, which shows M_{DD} (or M_{DIV} in Schaufele) as function of C_L, Λ, and $(t/c)_{av}$. In these charts M_{DD} is a linear function of C_L, so it is relatively easy to establish the relationship. The definition of M_{DIV} is unstated for these charts, but Schaufele Fig. 12-10 shows a value of M_{DIV} for which the drag rise is 16 counts. From this figure, $M_{DD_{Douglas}} - M_{DIV} = 0.015$, so when using this method, this value should be added to the values in Schaufele Fig. 4-8 to obtain Douglas values.

Another alternative (and preferred) approach is to use the Korn equation (Ref. 12.6.10.5) to determine M_{DD}. This approach will therefore be used in this paper.

$$M_{DD} = \frac{\kappa_a}{\cos \Lambda_{c/2}} - \frac{\frac{t}{c}}{cos^2 \Lambda_{c/2}} - \frac{C_l}{10\ cos^3 \Lambda_{c/2}} \qquad (12.5.10.2)$$

where

M_{DD} = wing drag divergence Mach number (Douglas definition)
κ_A = airfoil technology factor
C_l = wing section lift coefficient
$\Lambda_{c/2}$ = wing sweep at mid-chord

For a wing with different airfoil sections at different spanwise locations, Ref. 12.5.10.5 suggests that this equation should be applied to each wing spanwise section with the same airfoil section, and values of M_{DD} for all spanwise sections should be averaged. During conceptual design, this level of detail may well not be known, so taking the average values of t/c and $\Lambda_{c/2}$ for the whole wing is an approximation to the more exact method, and can be used for this analysis. The mid-chord is selected for defining wing sweep because the upper surface shock is approximately at that location. This method is preferred because it contains a technology factor (κ_A) so that the equation can be applied to non-supercritical and supercritical wing sections. The effect of increasing κ_A can also be evaluated. Reference 5 suggests a value of $\kappa_A = 0.87$ for a NASA 6-series section, and 0.95 for a supercritical wing. This value will increase as supercritical wing sections are further developed. This equation correlates reasonably well in the operational area of interest (C_L = 0.5 to 0.6 and $\Lambda_{c/2} = 25^0$ to 35^0) with Schaufele, Fig.4-8.

Sweep at any fraction of the chord may be calculated using the equation

$$\tan \Lambda_x = \tan \Lambda_{LE} - \frac{4x\,(1 - \lambda)}{A(1 + \lambda)} \qquad (12.5.10.3)$$

where

Λ_x = wing sweep at fraction x of chord
Λ_{LE} = wing sweep at leading edge
x = fraction of wing chord
A = wing aspect ratio

λ = wing taper ratio

The incompressible drag polar is calculated conventionally using the equation

$$C_{D_{incomp}} = C_{D_0} + \frac{(C_L)^2}{\pi\, AR\, e} \qquad (12.5.10.4)$$

where

$C_{D_{incomp}}$ = incompressible drag coefficient

C_{D_0} = incompressible zero lift drag coefficient

AR = aspect ratio

e = Oswald efficiency factor based on symmetric polar

Values of C_{D_0} and e can be calculated using standard textbook methods. The total drag coefficient is therefore the sum of $C_{D_{incomp}}$ and C_{D_C}. From a knowledge of the aircraft incompressible drag polar, plus the drag rise assumptions described above, it is possible to generate a drag map.

12.5.10.2 Generation of Derivative Plots

This plot enables other plots to be generated; in particular:

- the compressible flow drag polar (Fig. 12.5.10.7)
- a plot of L/D as a function of C_L and Mach number (Fig. 12.5.10.8)
- a plot of ML/D as a function of C_L and Mach number (Fig. 12.5.10.9).

This last set is critically important in telling the designer the maximum value of ML/D that can be obtained, and the associated Mach number. Each of these plots was generated using Excel, but other software, such as MATLAB, could equally well have been used.

For these plots a value of κ_A = 0.925 was used. Selecting an appropriate value for κ_A is difficult for supercritical wings, because airfoil technology is constantly improving. Reference 3 (published in 2010) suggests a value of 0.95 for supercritical wings. A rough rule of thumb is to assume that the value of κ_A increases by 0.005 every decade.

This procedure does not include compressibility effects on fuselage and empennage drag. Raymer (Ref.12.5.10.1, Fig. 12.31) shows fuselage M_{DD} in terms of the fuselage length and diameter, and for a typical widebody airliner, the fuselage M_{DD} is above that of the wing, even at low C_L. However, comparison with aircraft drag polars shows an underestimation of drag creep prior to divergence, a feature of many modern wing designs. Drag creep may also be due to the need to modify wing root sections to minimize wing/fuselage interference effects. Obert (Ref. 12.5.10.4) has extensive examples and discussion on this subject.

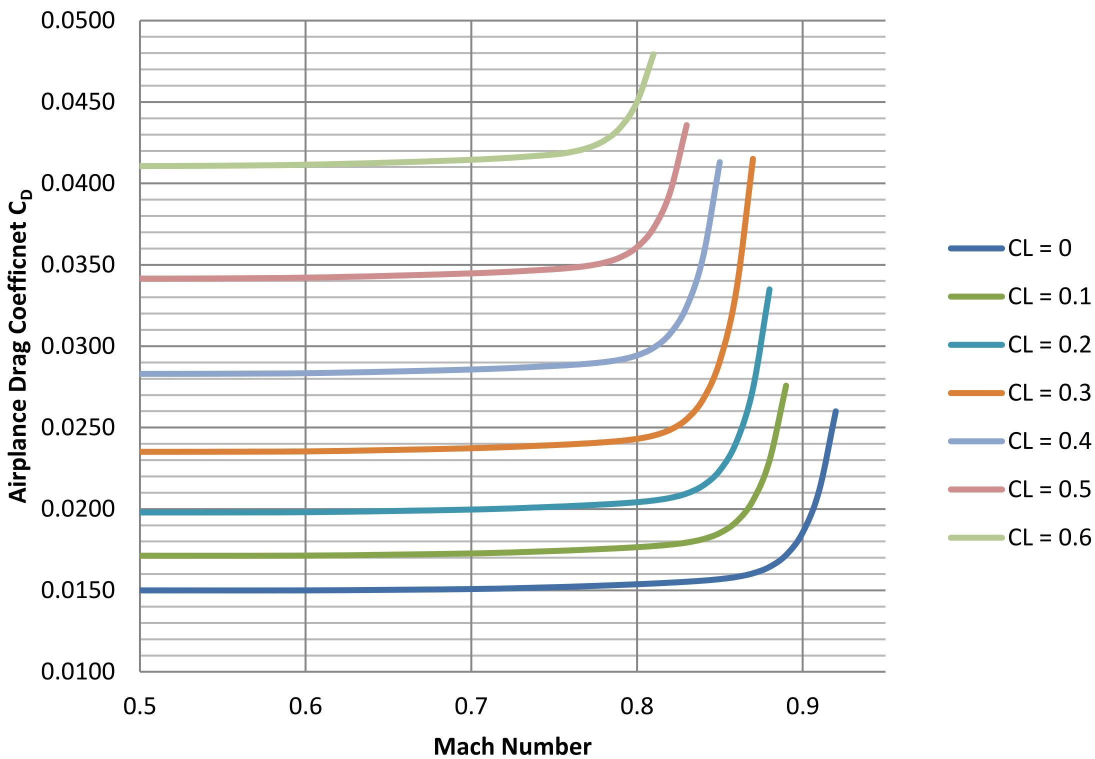


Figure 12.5.10.6 Excel-generated drag map for typical airliner

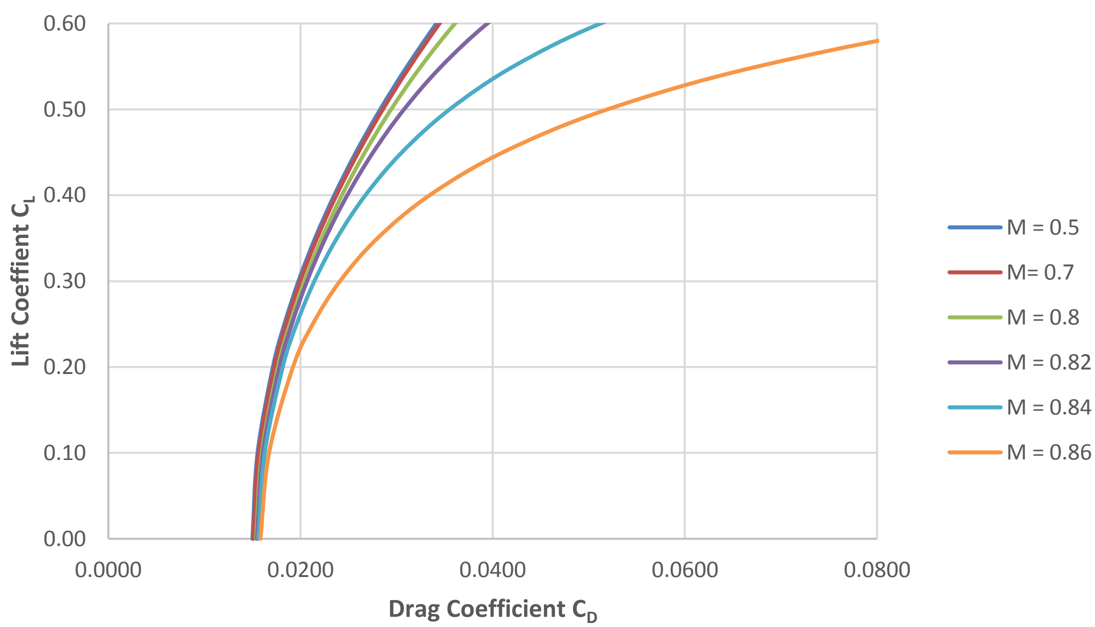


Figure 12.5.10.7 Excel-generated drag polars for a typical airliner

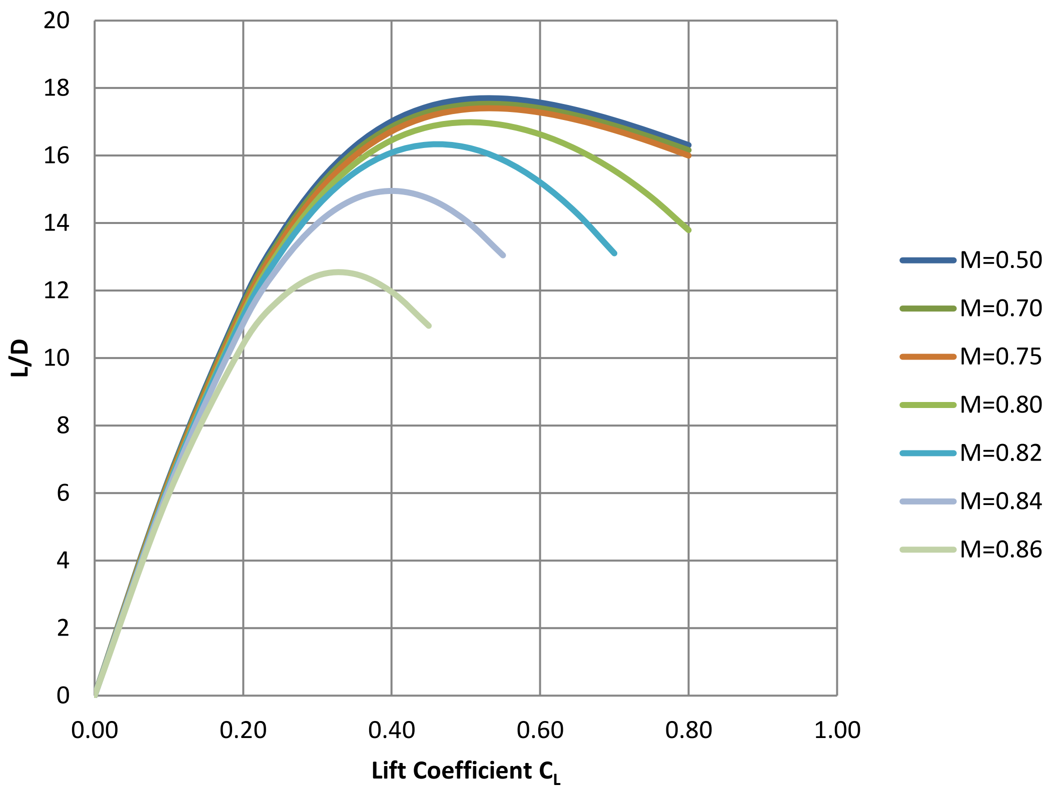


Figure 12.5.10.8 Excel-generated plot of L/D vs. C_L for typical airliner

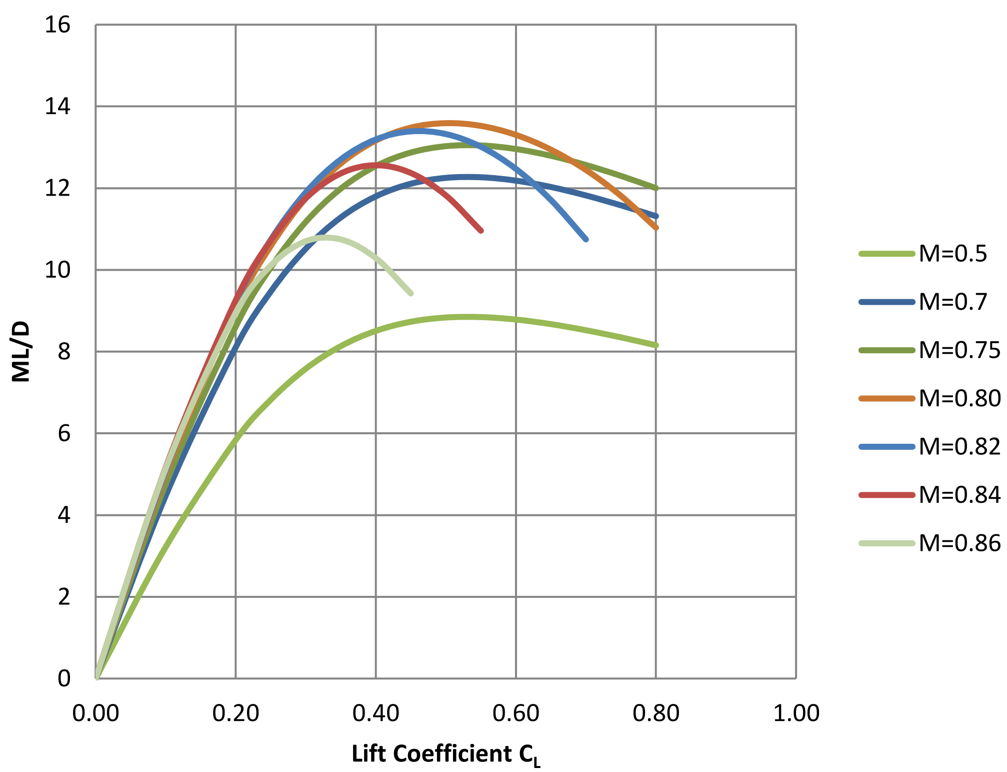


Figure 12.5.10.9 Excel-generated plot of ML/D vs. C_L for typical airliner

A spreadsheet is available online at www.adac.aero that was used to generate some of the figures in this paper:

Drag Map + Polars + LoD vs CL + MLoD vs CL using Korn rev 8.0.xlsx.

This spreadsheet produced Figures 12.5.10.5 through 12.5.9. It can easily be modified for changes in airplane geometry, value of technology factor, or shape of drag rise.

12.5.10.3 Conclusions

The generation of a drag map is a critical part of defining aerodynamic characteristics of a configuration that operates in the high subsonic region. It is important for students to understand the significance of the experimentally-generated drag map (from wind tunnel or CFD data), and also to know how a representative drag map and derivative plots can be generated at the conceptual design level. In particular the plot of *ML/D* as a function of C_L and *M* suggests the optimum cruise values for these variables. This process appears only in Schaufele's textbook (Ref. 6), from which these procedures were drawn. The major deviation from Schaufele's method is that whereas he used a fixed relationship between C_L, Λ, *t/c* and M_{DD}, this method uses the Korn equation, which includes a technology factor, κ_a, to reflect the level of technology in the airfoil section. Both design and technology variables can be perturbed and the effect of these changes evaluated.

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