5.3.11 Climb and Glide

For commercial aircraft, two climb requirements usually constrain the design space in *T/W* and *W/S*. These are engine-out climb immediately after takeoff (often referred to as second-segment climb), and the climb requirement at start of cruise.

Second Segment Climb

Second segment climb requirements are discussed in detail in the annotation to section 17.3.1. The following description is an abbreviated version to enable a first estimate of the lower bound of *T/W*.

For second-segment climb, Raymer Eq. (5.28) must be modified to account for the engine-out condition. For an airplane with *n* engines, the climb requirement becomes:

$$\left(\frac{T}{W}\right)_{2nd\,seg} = \frac{n}{n-1}\left(\frac{1}{\left(\frac{L}{D}\right)_{2nd\,seg}} + G\right) \tag{5.3.11.1}$$

The climb requirement, G, can be obtained from Raymer Table F.4 (note the typographical error in the left-hand column for "Minimum climb gradient for aircraft with *n* engines" should be "*n*=2"). Thrust is a weak function of speed, and second segment speed is a function of wing loading, so the value of $(T/W)_{ref}$ will also be a weak function of wing loading, as illustrated in the example below. However, for the purpose of this analysis, $(T/W)_{ref}$ will be assumed to be independent of wing loading.

The value of *L/D* at takeoff may not be the optimum. The aircraft will normally be using takeoff flaps, which enables the aircraft to take off at a lower speed, but adds to the zero-lift drag. In addition, the aircraft may be flying at a C_L that is greater than that for optimum *L/D*.

As a first estimate, assume $\left(\frac{L}{D}\right)_{2nd\,seg} = 0.75\left(\frac{L}{D}\right)_{\max}$ (5.3.11.2)

The value of $(L/D)_{2nd\,seg}$ will be refined when more information is known about the aircraft in Chapter 17. The value of $(L/D)_{max}$ can be calculated from the drag polar (if known), or estimated from methods in Raymer Section 3.4.4.

Finally, the value of *T/W* at the second segment condition must be factored by the ratio of reference thrust to thrust at that condition, i.e.,

$$\left(\frac{T}{W}\right)_{ref} = \left(\frac{T}{W}\right)_{2nd\,seg}\left(\frac{T_{ref}}{T_{2nd\,seg}}\right) \tag{5.3.11.3}$$

For example, for a twin-engine airplane, for which $(L/D)_{max}$ =13.5, Raymer Table F.4 shows that the required second segment climb gradient is 0.024.

Assume
$$\left(\frac{L}{D}\right)_{2nd\,seg} = 13.5 \times 0.75 = 10.1$$

From equation (5.3.11.1)
$$\left(\frac{T}{W}\right)_{2nd\,seg} = 2\left(\frac{1}{10.1} + 0.024\right) = 0.246$$

This value of T/W is the value at during the second segment climb. This value must now be factored by the ratio of the reference thrust (sea level static), to the thrust during second segment climb, as shown in equation (5.3.11.3). The aircraft speed during second segment climb must be estimated. This is only an approximation, because thrust is a relatively weak function of speed.

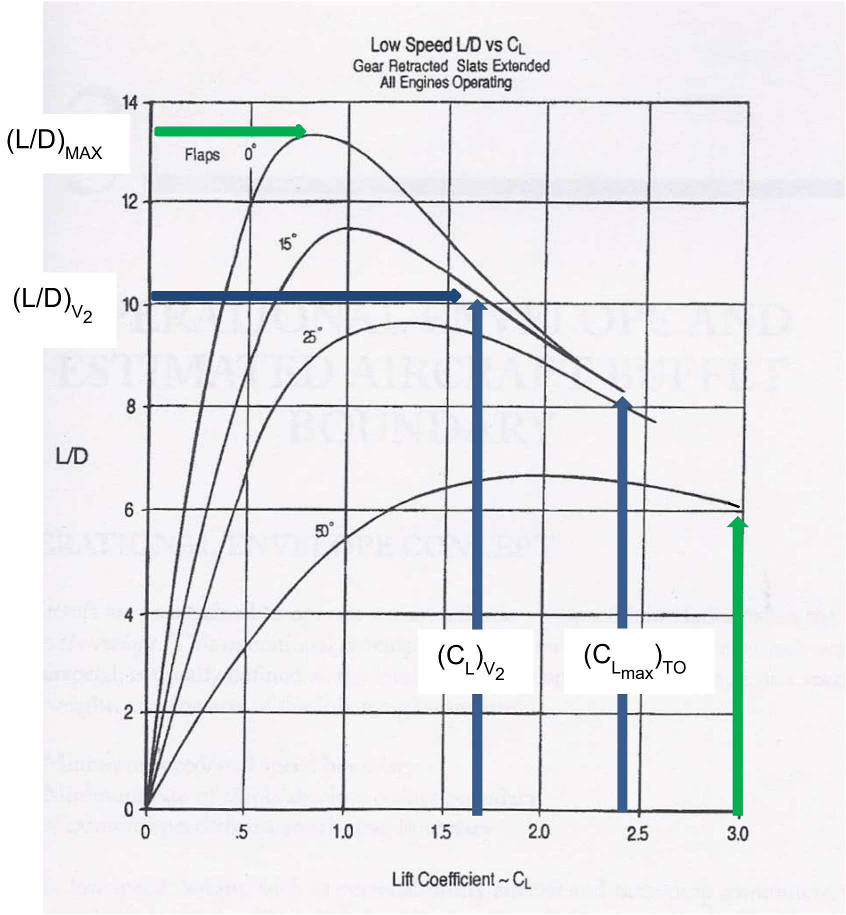

Fig. 5.3.11.1 Example of L/D vs. C_L plot

For example, for a wing with 30° sweep and triple-slotted flap and slat, Raymer Fig 5.3 shows $C_{L_{max}} = 3.0$

From Raymer page 127 (for 6th Edition, or page 128 for 5th Edition), assume $\left(C_{L_{max}}\right)_{2nd\,seg} = 0.8\,C_{L_{max}} = 2.4$ (this happens to be the case for the DC-9 shown in this example)

From Raymer Table F.2 $V_{2nd\,seg} = V_2 = 1.2\left(V_{stall}\right)_{takeoff\,flaps}$

so $\left(C_L\right)_{2nd\,seg} = \dfrac{\left(C_{L_{\max}}\right)_{2nd\,seg}}{1.2^2} = \dfrac{2.4}{1.44} = 1.67$

From Raymer Table 5.5, typical wing loading for a jet transport is 120 lb/ft^2 (this is where we assume that *(T/W)$_{ref}$* is not a function of *(W/S)$_{ref}$*),

so dynamic pressure at liftoff, $q = \dfrac{\frac{W}{S}}{\left(C_L\right)_{2nd\,seg}} = \dfrac{120}{1.67} = 72\ \dfrac{lb}{ft^2}$

Approximately $\dfrac{q}{M^2} = 0.7\,p$, and sea level air pressure is 2116 lb/ft^2, so at sea level

$$M_{2nd\,seg} = \sqrt{\frac{72}{2116 \times 0.7}} = 0.22$$

From Raymer Table E.2, for a high bypass ratio turbofan (BPR=8.0) at full thrust,

$$\frac{T_{M=0.22}}{T_{ref}} = 0.68$$

so $\left(\dfrac{T}{W}\right)_{ref} = \left(\dfrac{T}{W}\right)_{2nd\,seg}\left(\dfrac{T_{ref}}{T_{2nd\,seg}}\right) = \dfrac{0.246}{0.68} = 0.36$

Climb speeds immediately after takeoff (which are the critical design conditions) are specified by the FARs. This issue is discussed in the annotations to section 17.3.1. Fortunately the specified climb speeds are a function of stall speed, which is in turn a function of wing loading for a specified wing design. Although the pilot may not be able to select the optimum *q*, the point on the generalized *T/W* vs. *W/S* curve remains the same, so that the climb *T/W* required to satisfy the FARs is still independent of *W/S*.

Start of Cruise

For *T/W* at start of cruise, Eqs. (5.28) and (5.29) can be combined as

$$\left(\frac{T}{W}\right)_{cruise} = \left(\frac{D}{W}\right)_{cruise} + G = \frac{q\,C_{D_0}}{\left(\frac{W}{S}\right)_{cruise}} + \left(\frac{W}{S}\right)_{cruise}\frac{1}{q\pi Ae} + G \qquad (5.3.11.4)$$

This is an expression for *T/W* as a continuous function of *W/S*, and may be used as a constraint line for cruise capability with a specified climb gradient potential, *G,* and dynamic pressure, *q*. Calculate *q* based on selected cruise Mach number (e.g. Mach 0.82) and altitude at start of cruise, which is typically around 35,000 ft. Very long range (transpacific) transports may start at around 31,000 ft. To be conservative, the weight is normally assumed to be the takeoff gross weight, even though some fuel has been burned in taxi, takeoff and climb, so the value of *W/S* in this equation is the reference value. The engine thrust at start of cruise is much less than the reference value, so the *T/W* correction must still be applied.

The climb requirement is usually that at the operational ceiling (see Table 5.3.11.1 below). The required climb gradient, *G,* is the required climb rate divided by forward velocity.

Definition	Climb Rate Requirement [ft/min]
Absolute Ceiling	0
Service Ceiling	100
Operational Ceiling	300
Combat Ceiling	500

Table 5.3.11.1 Ceiling Requirements

Eq. (5.3.11.4) assumes that the airplane is neither accelerating nor slowing down. In practice, optimum climb conditions require that the airplane either accelerate (when climbing at a constant IAS), or slow down (when climbing at a constant Mach number). This requires a correction to the value of *T/W* which is effected by dividing the required value of *G* in the equation above by the K.E. Factor. This correction is described in the Annotation to Section 17.3.1.

Minimum Value of T/W

As a point of interest, the wing loading at which the required *T/W* is a minimum may be found by taking the differential of Eq. (5.3.11.3) with respect to *W/S* and setting to zero:

$$\frac{d\left(\frac{T}{W}\right)}{d\left(\frac{W}{S}\right)} = -\frac{qC_{D_0}}{\left(\frac{W}{S}\right)^2} + \frac{1}{q\pi A\, e} \tag{5.3.11.5}$$

Rearranging terms:

$$\left(\frac{W}{S}\right)_{\min\frac{T}{W}} = q\sqrt{C_{D_0}\,\pi A\, e} \tag{5.3.11.6}$$

At the cruise condition for a subsonic transport, the value of $(W/S)_{min\ T/W}$ is generally above that of the landing constraint in Figure 5.4.1.

We could then put this expression for *W/S* back into Eq. (5.3.11.4) and obtain Raymer's Eq. (5.31) which is

$$\frac{T}{W} = G + 2\sqrt{\frac{C_{D_0}}{\pi\, A\, e}} \qquad (5.31)$$

This expression is only valid for the condition that the airplane is flying at a value of q for which T/W is a minimum.

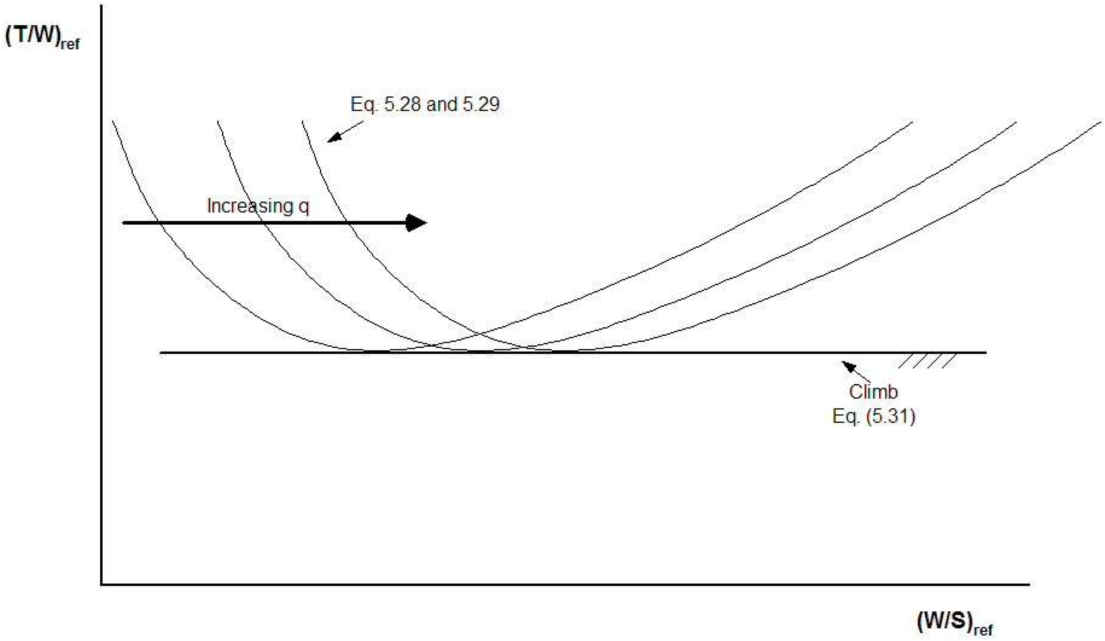

Fig. 5.3.11.1 Required *T/W* for climb as a function of *W/S* and *q*

For small climb angles (which is usually the case for commercial airplanes) we can assume that $L = W$, and Eq. (5.28) can simply be written as

$$\frac{T}{W} = G + \frac{1}{\left(\frac{L}{D}\right)_{climb}} \qquad (5.3.11.7)$$

This is true for all values of q, but it is only true as the minimum value of T/W provided that L/D is maximized, which can be achieved through the appropriate selection of q.

Notice that for the condition of $(L/D)_{max}$ then

$$\left(\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{\frac{C_{D_o}}{\pi A\, e}}} \qquad (3.4.2)$$

This relationship was established in the annotations to Section 3.4.4.