

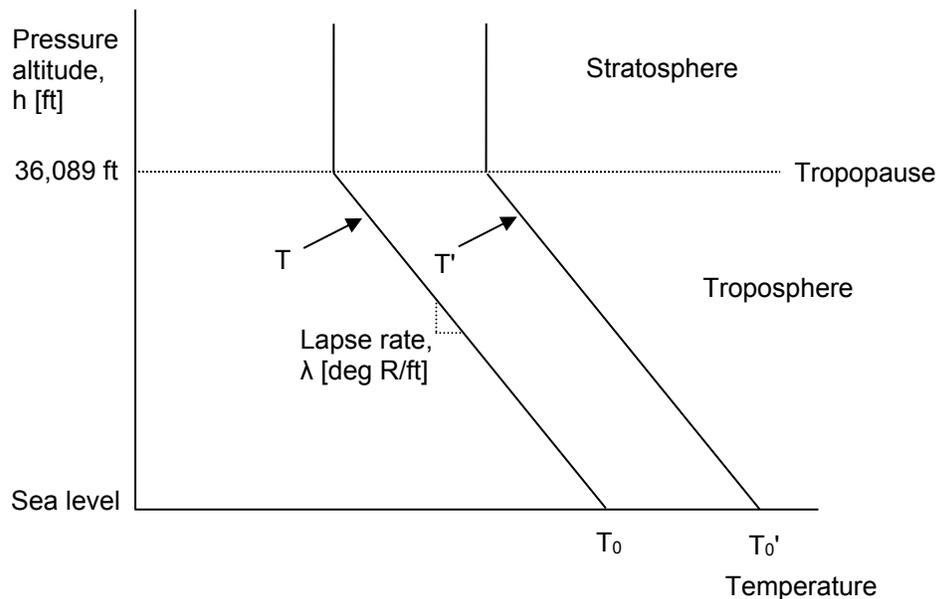
## Appendix B Standard Atmosphere

For the purposes of analyzing aircraft performance, the characteristics of the atmosphere can be simplified into a standard model. This section discusses the characteristics of the atmosphere up to the top of the stratosphere (approximately 65,600 ft). The aircraft performance engineer must know the values of atmospheric temperature,  $T$ , pressure,  $p$ , density,  $\rho$ , and the local speed of sound,  $a$ , throughout the aircraft's flight.

Equations are given here without derivation. For derivation the student should refer to Lan & Roskam (Ref 2).

### International Standard Atmosphere (ISA)

The most commonly used atmosphere model is the International Civil Aviation Organization (ICAO) Standard Atmosphere, of which the main part is the International Standard Atmosphere (ISA). At all altitudes of interest to an aircraft performance engineer, the ISA model is the same as that of the 1976 U.S. Standard Atmosphere, and the MIL-STD Standard Atmosphere.



ISA assumes the following characteristics:

Temperature lapse rate	$\lambda$	-0.003566 $^{\circ}\text{R}/\text{ft}$
Gas constant	$R$	53.35 $\text{ft}^{\circ}\text{R}$
Sea level pressure	$p_0$	2116.2 $\text{lb}/\text{ft}^2$
Sea level temperature	$T_0$	518.7 $^{\circ}\text{R}$ (= 59 $^{\circ}\text{F}$ )
Sea level density	$\rho_0$	0.002377 $\text{slugs}/\text{ft}^3$
Gravitational constant	$g$	32.17 $\text{ft}/\text{sec}^2$
Specific heat ratio	$\gamma$	1.4

The altitude used in most performance calculations is not the geometric altitude but the pressure altitude. The performance engineer is usually interested in the air pressure, density and temperature in the immediate vicinity of the airplane. Absolute altitude (except for calculations for time to climb) is of secondary importance.



Source: FAA AC 61-23B

Figure 1. Altimeter

An aircraft altimeter does not directly measure altitude. It is an aneroid barometer (Figure 1), and the dial converts the pressure value into an equivalent altitude. If the sea level pressure is not the standard value (as a result of high or low pressure weather systems), the altimeter must be corrected manually (using a knurled knob) to take account of the difference at that locality and at that time. The foregoing procedure applies below 18,000 ft MSL. Above that altitude, the altimeter is set to standard day pressure (29.92 inches of mercury) and pilots fly to the pressure altitude for a standard day.

### ***Troposphere***

At any altitude  $h$  [ft] within the troposphere, the temperature is given by:

$$T = T_0 + \lambda h \quad (1)$$

The pressure is:

$$p = p_0 \left( 1 + \frac{\lambda h}{T_0} \right)^{5.2561} \quad (2)$$

The density is derived from the standard gas equation:

$$\rho = \frac{p}{gRT} \quad (3)$$

Sometimes you may see reference to “density altitude”, which is the altitude on a standard day which has the same air density as that actually experienced. Thus a higher

value of density altitude implies that the air is less dense than that of a standard atmosphere at the same pressure altitude. Both aircraft and engine performance are degraded as the air density decreases.

The local speed of sound is a function only of temperature:

$$a = \sqrt{\gamma g R T} \quad (4)$$

### **Stratosphere**

In the stratosphere the temperature is independent of altitude and has a value of 390<sup>0</sup> R or -69.7<sup>0</sup> F.

Pressure is given by

$$\frac{p}{p_0} = 0.2234 e^{-\frac{h - 36,089}{20806.7}} \quad (5)$$

Density can be derived from the standard gas equation shown above.

### Non-Standard Atmosphere

There are many non-standard atmospheres, of which the most simple involves a temperature offset from the standard day temperature gradient, as shown in the figure above. On a hot day the whole atmosphere is displaced upwards, so that the geometric height of the tropopause is increased. For example, at ISA + 15 <sup>0</sup>C (+ 27 <sup>0</sup>F) the geometric height of the tropopause increases by about 2,200 ft. At a given geometric altitude (except at sea level) air pressure increases, because there is now more atmosphere bearing down on a given area. There is no correction for ambient temperature on a standard altimeter so pilots must be aware that on a cold day the geometric altitude of an airplane will be lower than the pressure altitude as indicated on the altimeter. For example, on an ISA - 10 <sup>0</sup>C (- 18 <sup>0</sup>F) day, an altimeter indicating 6000 ft will actually be at 5787 ft MSL.

Fortunately the vertical displacement of the atmosphere doesn't concern us (and thus makes calculations a lot easier), because we are concerned only with pressure altitude, and on a pressure altitude scale (as shown on the figure above), the height of the tropopause remains at a pressure altitude of 36,089 ft.

### **Troposphere**

The temperature offset is defined by:

$$T' = T + \Delta T \quad (6)$$

so that the temperature at altitude  $h$  is:

$$T' = T_0' + \lambda h \quad (7)$$

The assumed pressure variation with altitude is unchanged (remember that the altitude is defined as the pressure altitude, not geometric altitude).

The density is therefore:

$$\rho' = \frac{p}{gRT'} \quad (8)$$

where the pressure,  $p$ , is obtained from Eq. (2) above.

### **Stratosphere**

Temperature in the stratosphere is given by:

$$T' = 390 + \Delta T^0 R \quad (9)$$

Pressure, by definition, remains independent of temperature offset from an ISA day and is therefore defined by Eq. (5). Density as before can be calculated from Eq. (8).

There are many other non-standard atmospheres, some of which are defined by MIL-STDs. A review of some of these atmosphere models may be found in Ref 1.

### Effect of humidity

ISA assumes dry air. Water vapor has a lower molecular weight than either nitrogen or oxygen, so humid air is less dense than dry air at the same temperature and pressure. According to Roskam (Ref 2), the density of saturated air is 1.8% less than that of dry air. This is small but significant. However, for the purposes of performance calculations this effect is usually neglected.

### Absolute Viscosity

For skin friction calculations, Reynolds Number must be calculated, which implies that absolute viscosity must be known. This is a function only of temperature and may be determined from the equation from Ref. 2:

$$\mu = \left[ 0.3170 (T^0 R)^{\frac{3}{2}} \left( \frac{734.7}{T^0 R + 216} \right) \right] \times 10^{-10} \frac{lb \text{ second}}{ft^2} \quad (10)$$

References:

1. Zuppardo, Joseph S., “ Graphical Comparison of U.S. Standard Atmospheres and Military Standard Climatic Extremes”, Wright Patterson AFB Air Force Materiel Command Dept ASC/ENFTA, Report ASC-TR-93-5002, Feb 1993.
2. Lan, Chuan-Tau, and Roskam, Jan, “Airplane Aerodynamics and Performance”, Roskam Aviation and Engineering, 1981.