12.5.10 Transonic Parasite Drag

The title of this section is somewhat of a misnomer, because there is a significant component of drag due to lift in this definition. Modern supersonic fighters have a significant margin of transonic thrust minus drag, so this topic is of somewhat less interest than in the early days of supersonic flight, when the thrust margin was very small. For designers of commercial airliners, this topic is of overarching importance. The measure of aerodynamic efficiency of an aircraft is defined in the Breguet range equation as *ML/D*. Specific fuel consumption is reasonably independent of speed, so that for a given structural weight fraction, the designer wants to maximize *ML/D*. Raymer Fig. 12.33 illustrates the difficulty. An enormous amount of time and effort is expended in increasing drag divergence Mach number by an amount as small as 0.01. Billions of dollars in aircraft sales (as exemplified by the ongoing battle between the 737 and A320) can depend on which airplane has the higher value of *ML/D*.

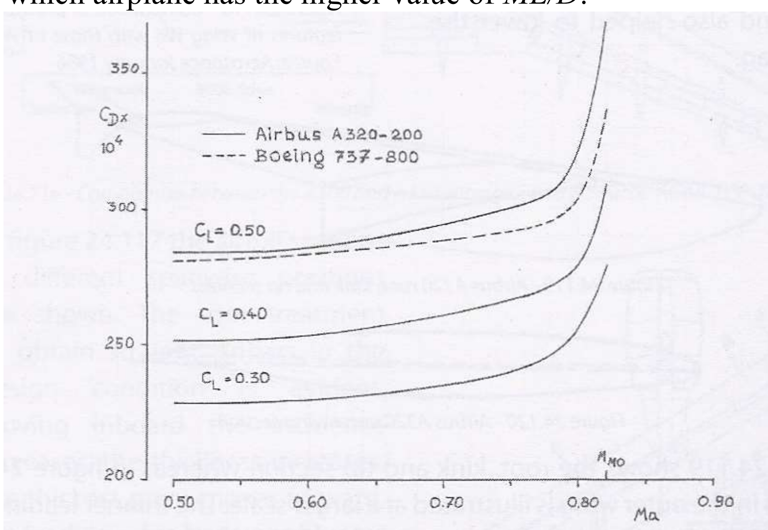

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Fig. 12.5.10.1 Drag Rise Comparison Between B.737 and A320

A useful way to define the transonic characteristics of an aircraft is by means of the drag map (Fig. 12.5.10.1) , which shows the drag rise (increase in compressibility drag, C_{D_C}) with increasing Mach number for different lift coefficients, in the region of the drag divergence Mach number, M_{DD}. For a given *t/c* and Λ, the aerodynamicists in the design team are trying to move the curves to the right. From a knowledge of the drag map, the optimum value of *ML/D* can be derived analytically, as described later in this Annotation.

Unfortunately, the lack of industry consensus on definitions adds to the difficulty of comparing values. M_{crit} is the Mach number at which compressibility drag first appears. $M_{DD_{Boeing}}$ is the Mach number at which compressibility drag reaches 20 counts. Raymer suggests that $M_{DD_{Boeing}} - M_{crit} = 0.080$. This is not an exact relationship, and is a function of wing design. For example, in the figure above for the A320, $M_{DD_{Boeing}} = 0.775$ and $M_{crit} = 0.575$, yielding a difference of $\Delta M = 0.200$. $M_{DD_{Douglas}}$ is the Mach number where the gradient of the C_{D_C} vs. Mach curve has a gradient of 0.10. Raymer also suggests that

$M_{DD_{Douglas}} - M_{DD_{Boeing}} = 0.060$. For the A320, $M_{DD_{Douglas}} = 0.795$, yielding a difference of $M_{DD_{Douglas}} - M_{DD_{Boeing}} = 0.020$. The shape of the A320 drag rise is fairly typical for commercial transports with supercritical wing section, although there are differences between aircraft types. For the DC-9-30, which did not have a supercritical wing section, $M_{DD_{Douglas}} - M_{DD_{Boeing}} < 0.010$ (Ref. 12.5.10.1). The Douglas definition has gained greater acceptance, and if a value is stated without its definition, it can generally be assumed to be the Douglas value.

To generate a drag map using a spreadsheet, the relationship between C_{D_c} and $(M\text{-}M_{DD})$ must be expressed algebraically. A typical shape which may be used to plot a drag map is:

$$C_{D_c} = 0.05\left(\frac{(M - M_{DD}) + 0.3}{0.352}\right)^{30} + 0.017\left((M - M_{DD}) + 0.3\right)^{2} \qquad (12.5.10.1)$$

This function is not valid when $(M\text{-}M_{DD}) < -0.3$, so the value of C_{D_C} must be set to zero for that condition in a spreadsheet using an Excel conditional function:

IF(condition, value_if_true, value_if_false)

It will also only be valid for Mach numbers up to about $M_{DD} + 0.04$.

Eq. (12.5.10.1) is plotted in Fig 12.5.10.2 and compared with the drag rise curve due to Schaufele (Ref 12.5.10.2). Both of the curves are reasonable fits to the drag rise of commercial aircraft. Note that the value of M_{DD} on Schaufele's curve is about the same for both Boeing and Douglas definitions, although for this particular curve the drag divergence Mach number (M_{DIV}) is defined as the value for which $C_{D_C} = 0.0016$. Schaufele's curve only exists in a tabular format, so the equation above is preferred for generating a drag map.

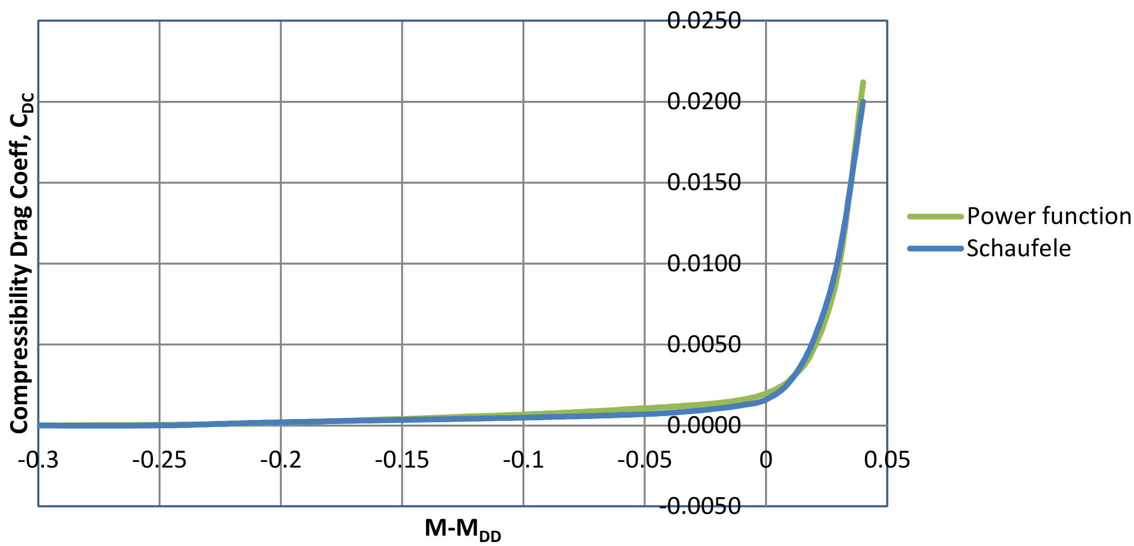

Fig. 12.5.10.2 Drag rise comparison, C_{D_C} vs. $(M\text{-}M_{DD})$

In Section 12.5.10 Raymer describes a method for determining the value of M_{DD} (Boeing definition) for a given wing design. This includes adjustments to M_{DD} for different values of C_L and t/c (Raymer Fig. 12.30). Provided that the shape of the drag rise is known (or assumed), it is therefore possible to generate a drag map based on the methods in this section. However, Raymer's method is not amenable to the use of MATLAB or a spreadsheet.

An alternative is to use Schaufele Fig. 4-8, which shows M_{DD} (or M_{DIV} in Schaufele) as function of C_L, Λ, and $(t/c)_{av}$. In these charts M_{DD} is a linear function of C_L, so it is relatively easy to establish the relationship. The definition of M_{DIV} is unstated for these charts, but Schaufele Fig. 12-10 shows a value of M_{DIV} for which the drag rise is 16 counts. From this figure, $M_{DD_{Douglas}} - M_{DIV} = 0.015$, so this value should be added to the values in Schaufele Fig. 4-8 to obtain Douglas values.

Another alternative (and preferred) approach is to use the Korn equation (Ref. 12.5.10.1) to determine M_{DD}.

$$M_{DD} = \frac{\kappa_A}{\cos\Lambda_{c/2}} - \frac{\frac{t}{c}}{\cos^2\Lambda_{c/2}} - \frac{C_L}{10\cos^3\Lambda_{c/2}} \qquad (12.5.10.2)$$

where

- M_{DD} = wing drag divergence Mach number (Douglas definition)
- κ_A = airfoil technology factor
- C_L = wing lift coefficient
- $\Lambda_{c/2}$ = wing sweep at mid-chord

This equation should be applied to a wing spanwise section, and values of M_{DD} for all sections should be averaged. Taking the average values of t/c and $\Lambda_{c/2}$ for the whole wing is an approximation to the more exact method. The mid-chord is selected for defining wing sweep because the upper surface shock is approximately at that location. The reference suggests a value of $\kappa_A = 0.87$ for a NASA 6-series section, and 0.95 for a supercritical wing. This value will increase as supercritical wing sections are further developed. This equation correlates reasonably well in the operational area of interest (C_L = 0.5 to 0.6 and $\Lambda_{c/2} = 25^0$ to 35^0) with the Fig. 12.1.1 in the annotation to section 12.1.

The Power function in Fig. 12.5.10.2 shows drag rise referenced to the Boeing definition, and for this particular drag rise curve:

$$M_{DD_{Boeing}} = M_{DD_{Douglas}} - 0.01 \qquad (12.5.10.3)$$

In order to use the drag rise plot of Fig. 12.5.10.2 with the Korn equation to generate a drag map, a correction of - 0.01 must therefore applied to the Korn equation of Eq. (12.5.10.2).

Sweep at any fraction of the chord may be calculated using the equation

$$\tan \Lambda_x = \tan \Lambda_{LE} - \frac{4x\,(1-\lambda)}{A\,(1+\lambda)} \qquad (12.5.10.4)$$

where

Λ_x = wing sweep at fraction x of chord
Λ_{LE} = wing sweep at leading edge
x = fraction of wing chord
A = wing aspect ratio
λ = wing taper ratio

From this information, plus a knowledge of the aircraft incompressible drag polar, it is possible to generate a drag map. This plot enables other plots to be generated; in particular, plots of *L/D* as a function of C_L and Mach number, and *ML/D* as a function of C_L and Mach number. This latter set is critically important in telling the designer the maximum value of *ML/D* that can be obtained, and the associated Mach number. These will be discussed in more detail in the Annotation to Raymer 17.2.5.

Examples of an Excel-generated drag map, *L/D* vs. C_L and *ML/D* vs. C_L, applicable to a DC-10 are shown in Figs. 12.5.10.3, 12.5.10.4, and 12.5.10.5. and may be compared with data in Shevell (Ref. 10.5.10.3, Figs. 12.14 and 15.16).

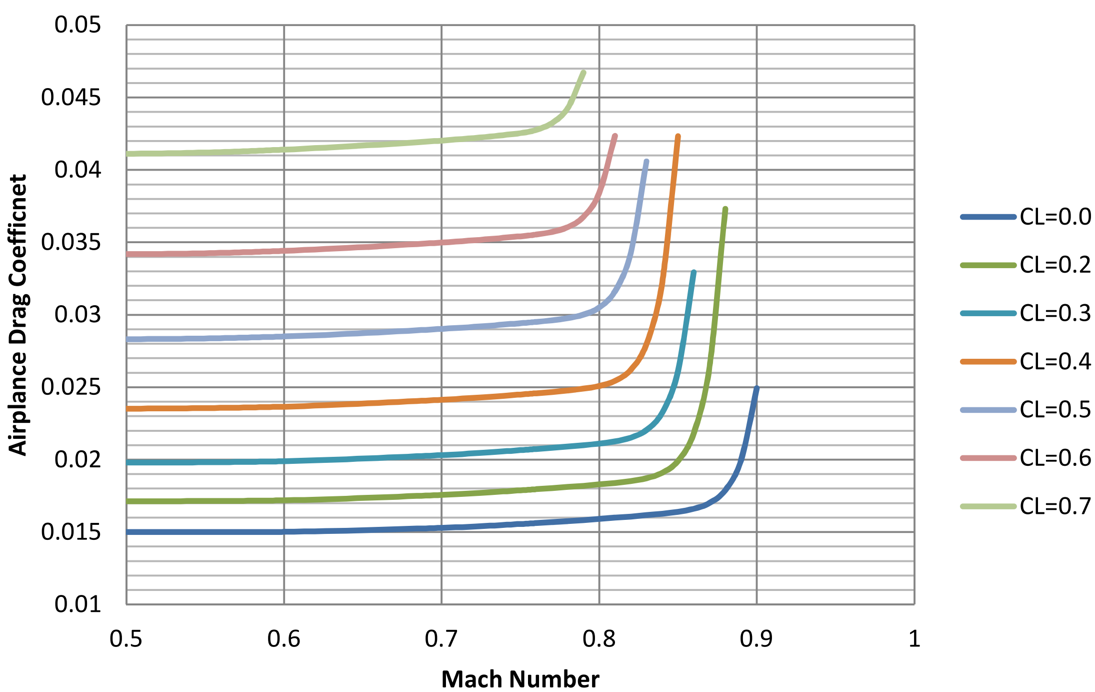

Fig. 12.5.10.3 Excel-generated drag map for DC-10

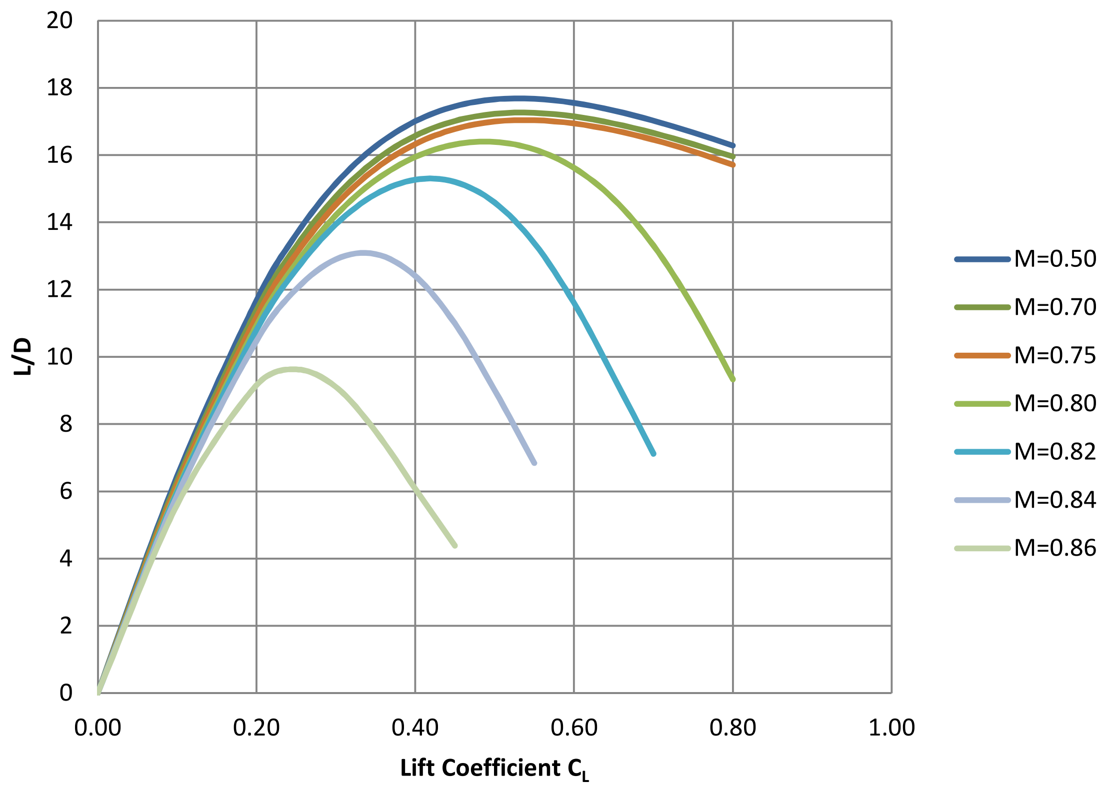

Fig. 12.5.10.4 Excel-generated plot of L/D vs. C_L for DC-10

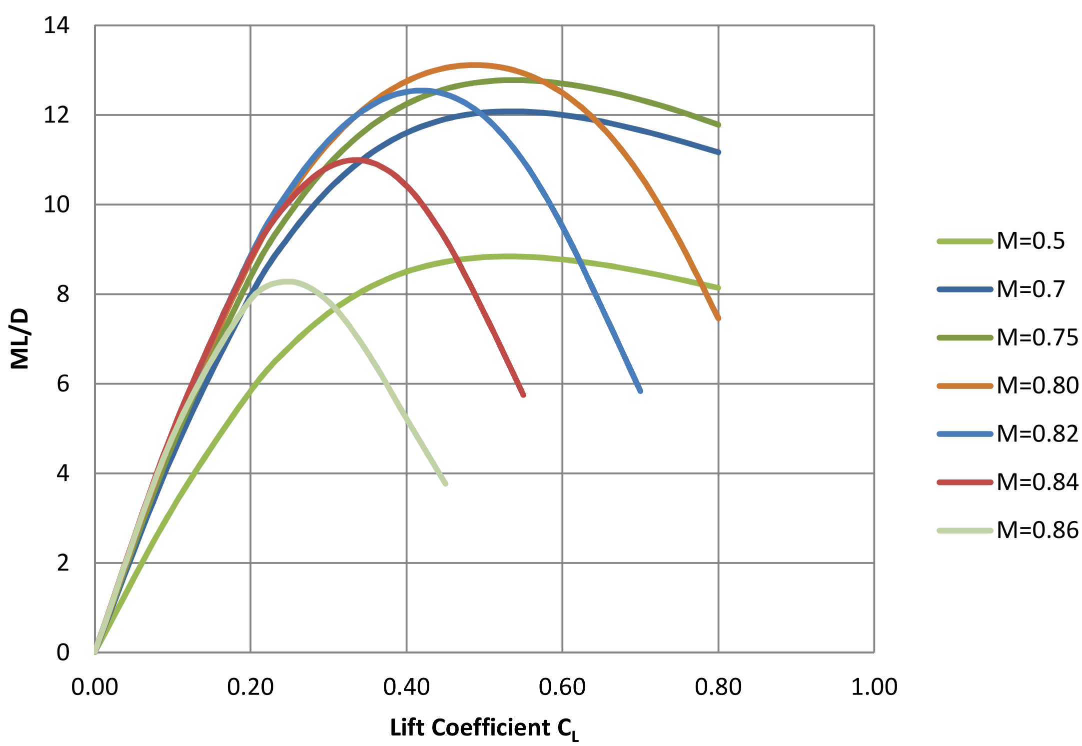

Fig. 12.5.10.5 Excel-generated plot of ML/D vs. C_L for DC-10

References

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12.5.10.3 Shevell, R.S., "Fundamentals of Flight", Prentice Hall, 1989