3.4.2 Mission Segment Fuel Fractions

The Breguet range equation (Raymer Eq. 3.5) is the cornerstone of payload-range calculations. It may be derived by assuming that the aircraft is in straight and level flight so that $T=D$ and $L=W$, and that aircraft speed (V), L/D, and specific fuel consumption (C) are constant. These are valid assumptions for a jet-powered aircraft.

The rate of change of aircraft distance with weight is (speed)/(fuel burn rate). The denominator has a negative value, because the weight of the aircraft is decreasing.

$$\frac{dR}{dW} = \frac{dR}{dt}\frac{dt}{dW} = \frac{V}{-CT} = \frac{V}{-CD} = \frac{V\left(\frac{L}{D}\right)}{-CW} = \frac{V\left(\frac{L}{D}\right)}{-C}\frac{1}{W} \qquad (3.4.2.1)$$

This may be integrated as follows:

$$R = \frac{V\left(\frac{L}{D}\right)}{C}\int_{W_{initial}}^{W_{final}} -\frac{dW}{W} = \frac{V\left(\frac{L}{D}\right)}{C}\left[-\ln W\right]_{W_{initial}}^{W_{final}}$$

$$\boxed{R = \frac{V\left(\frac{L}{D}\right)}{C}\ln\left(\frac{W_{inital}}{W_{final}}\right)} \qquad (3.4.2.2)$$

This is the Breguet range equation. To maximize range, the values of the product $V(L/D)$ and W_{inital}/W_{final} must be maximized, and C must be minimized.

To express the weight ratio as a function of other variables, the equation must be rearranged as:

$$\frac{W_{final}}{W_{initial}} = e^{-\frac{R}{\frac{V}{C}\left(\frac{L}{D}\right)}} \qquad (3.4.2.3)$$

This is the form in which the equation is used when calculating the total fuel required to perform a given mission.

Loiter Equation

A similar equation may be derived for the loiter condition. The rate of change of time with aircraft weight is simply 1/(fuel burn rate). Again, a negative sign shows that the rate of change of weight is negative. The change in endurance, *E*, with weight, *W*, is given by

$$\frac{dE}{dW} = \frac{1}{-CT} = \frac{1}{-CD} = \left(\frac{\frac{L}{D}}{-C}\right)\frac{1}{W}$$

This may be integrated, as before:

$$E = \int_{W_{initial}}^{W_{final}} \frac{1}{-C}\frac{L}{D}\frac{dW}{W} = \frac{1}{C}\frac{L}{D}\int_{W_{initial}}^{W_{final}} -\frac{dW}{W} = \frac{1}{C}\frac{L}{D}\left[-\ln W\right]_{W_{initial}}^{W_{final}}$$

$$E = \frac{1}{C}\frac{L}{D}\ln\left(\frac{W_{initial}}{W_{final}}\right) \qquad (3.4.2.4)$$

To maximize endurance, the values of *L/D* and $W_{initial}/W_{final}$ must be maximized, and *C* minimized.

To express the weight ratio as a function of other variables, the equation must be rearranged as:

$$\frac{W_{final}}{W_{initial}} = e^{-\left(\frac{E}{\frac{1}{C}\frac{L}{D}}\right)} \qquad (3.4.2.5)$$

This is the form in which the equation is used when calculating the total fuel burned in a mission that requires a loiter segment.

Selection of Cruise Lift/Drag Ratio

In Section 17.2.5 it will be shown that the optimum *L/D* for cruise is not at the maximum value of *L/D*. Maximizing range requires that the product *V(L/D)* in the Breguet range equation be maximized, and this occurs at a higher speed, and lower lift coefficient, than that for maximum *L/D*. The most common estimate is to use:

$$\left(\frac{L}{D}\right)_{cruise} = 0.866\left(\frac{L}{D}\right)_{max} \qquad (3.4.2.6)$$

This value should in the estimation of the available operating empty weight for the cruise portion of flight.

Maintaining Optimum L/D During Cruise or Loiter Segment

During the cruise or loiter segment, fuel is burned and the aircraft becomes progressively lighter. Flying at the optimum value of *L/D* requires that the value of C_L also be constant. From the definition of lift coefficient

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} \qquad (3.4.2.7)$$

it becomes apparent that as the numerator decreases, so must the denominator. Decreasing wing area is generally infeasible (although it has been tried on a few concepts), leaving the pilot the choice of decreasing either ρ or V. To maximize range, the product *V(L/D)* must be maximized, so slowing down is not an option. The alternative is to decrease ρ, which implies a gradual climb during cruise. This is called cruise climb, and can be seen in Raymer Fig. 3.2 Aircraft operating in controlled airspace (and this applies to most commercial aircraft operations) are restricted to fixed altitudes, separated by 1000 ft. increments above 18,000 ft. The aircraft must therefore perform a step-climb between permitted altitudes, as described in the Annotation to Section 17.2.4.

Occasionally an aircraft, such as a cruise missile, will be required to fly the cruise portion of flight at a fixed altitude. In this case it will have to be accepted that the aircraft will not be able to fly at a constant *L/D* but should still fly at the speed for which *V(L/D)* is maximized at that altitude.

For an airplane that is loitering it is often required to fly at a fixed altitude. In this case, slowing down is an option, because speed does not enter into the equation for loiter. The pilot should gradually pull back on the power levers to maintain the speed for maximum *L/D*.