19.3.4 Climb and Acceleration

The equation for the weight ratio required to achieve a given energy height (Eq. (19.8)) can be derived in much the same way as the Breguet range equation is derived. Starting from Eq. (19.94):

$$dh_e = -\frac{P_s}{CT}\,dW \qquad (19.3.4.1)$$

A negative sign is required because the airplane is losing weight as it gains energy height. Eq. (17.87) states that:

$$P_s = V\frac{(T-D)}{W}$$

So we can write:

$$\Delta h_e = \int_{i-1}^{i} \frac{V(T-D)}{-CT}\,\frac{dW}{W}$$

$$= \frac{V\left(1-\frac{D}{T}\right)}{-C}\log\left(\frac{W_i}{W_{i-1}}\right)$$

$$\log\left(\frac{W_i}{W_{i-1}}\right) = \frac{-C\Delta h_e}{V\left(1-\frac{D}{T}\right)}$$

$$\frac{W_i}{W_{i-1}} = \exp\left[\frac{-C\Delta h_e}{V\left(1-\frac{D}{T}\right)}\right] \qquad (19.8)$$

$$\text{where } \Delta h_e = \Delta\left(h + \frac{1}{2g}V^2\right)$$

The significance of this equation in the design of a hypersonic single-state-to-orbit vehicle is discussed in the annotation to section 21.3.1 Hypersonic Flight.