12.3 Aerodynamic Coefficients

Raymer mentions that aircraft drag is normally defined in terms of the number of "counts" of drag, which are the four digits to the right of the decimal point in the drag coefficient. It is good practice to quote all drag coefficients to exactly four significant figures. Unless some very refined drag comparisons are being made, there should be no need to quote a drag coefficient to more than four significant figures. It can also be confusing if they are quoted at less than four. So if the number of drag counts is 40, then the drag coefficient should be written as $C_D = 0.0040$.

Raymer Fig. 12.4 shows the difference in the shape of the drag polar for a wing with an uncambered and cambered airfoil. If a wing with an uncambered airfoil is set at some positive angle of incidence on a fuselage, which is often the case for transport aircraft, the same polar shape appears as for a cambered wing. This is due to the fact that when the wing is producing no lift, the fuselage will be at a negative angle of attack with respect to the airflow, and will contribute extra drag.

For moderate to highly cambered wings we must use Raymer's Eq. (12.5) which is

$$C_D = C_{D_{min}} + K\left(C_L - C_{L_{min\,drag}}\right)^2 \qquad (12.3.1)$$

Multiplying these terms out we get

$$C_D = C_{D_{min}} + KC_L^2 - KC_L C_{L_{min\,drag}} + KC_{L_{min\,drag}}^2 \qquad (12.3.2)$$

The gradient of this equation is

$$\frac{dC_D}{dC_L} = 2KC_L - KC_{L_{min\,\mathrm{drag}}} \qquad (12.3.3)$$

Note from the Annotation to Section 12.6.1 that the value of K which is derived from fitting drag polar experimental data to Eq. (12.3.1) will be significantly different from the value of K when fitting data to a symmetric polar.

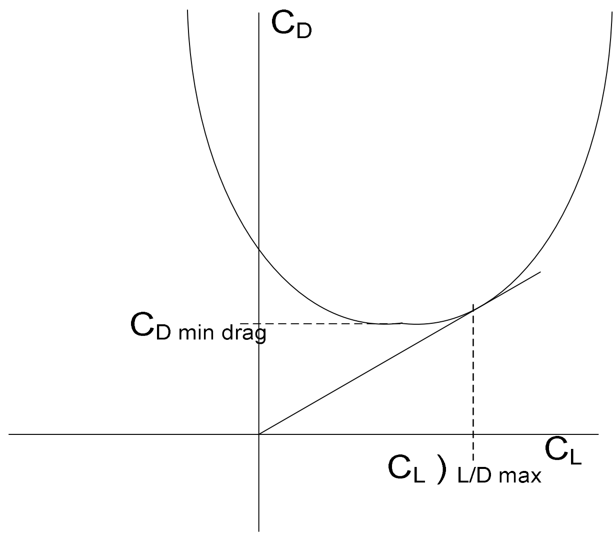

Fig. 12.3.3 Finding C_L at $(L/D)_{max}$ for a Cambered Airfoil

To find the conditions for *(L/D)max*, we want to find the point on the curve where the tangent from the origin touches the curve. That is where the gradient of the curve is equal to the gradient of the tangent. So we set

$$\frac{dC_D}{dC_L} = \frac{C_D}{C_L} \qquad (12.3.4)$$

Or

$$C_D = C_L \frac{dC_D}{dC_L} \qquad (12.3.5)$$

Combining Eq. (12.3.3) and (12.3.4)

$$C_D = KC_L^2 - KC_{L_{min\,drag}} C_L \qquad (12.3.6)$$

Combining Eq. (12.3.2) and (12.3.6)

$$C_{D_{\min}} + KC_L^2 - KC_L C_{L_{min\,drag}} + KC_{L_{min\,drag}}^2 = KC_L^2 - KC_L C_{L_{min\,drag}} \qquad (12.3.7)$$

From Eq. (12.3.2), when $C_L = 0$, then by definition $C_D = C_{D_0}$

$$C_{D_0} = C_{D_{min}} + KC_{L_{min\,drag}}^2 \qquad (12.3.8)$$

Substituting for $C_{D_{\min}}$ in Eq. (12.3.7)

$$C_{D_0} + KC_L^2 - KC_L C_{L_{min\,drag}} + KC_{L_{min\,drag}}^2 = 2KC_L^2 - KC_L C_{L_{min\,drag}} \qquad (12.3.9)$$

Simplifying Eq. (12.3.9) we get for the condition of maximum *L/D*

$$K C_L^2 = C_{D_0} + K C_{L_{min\,drag}}^2 \tag{12.3.10}$$

$$\left(C_L\right)_{max\frac{L}{D}} = \sqrt{\frac{C_{D_0}}{K} + C_{L_{min\,drag}}^2} \tag{12.3.11}$$

Note that for the condition for which $C_{L_{\min drag}} = 0$, Eq. (12.3.10) reduces to Raymer's Eq. (17.4).

We can then substitute the value of C_L at max *L/D* into Eq. (12.3.3) to obtain the value of maximum *L/D* for this condition.

$$\left(\frac{L}{D}\right)_{max} = \frac{1}{K\left(2\left(C_L\right)_{max\frac{L}{D}} - C_{L_{min\,drag}}\right)} \tag{12.3.12}$$

For the condition for which $C_{L_{\min drag}} = 0$

then $$\left(\frac{L}{D}\right)_{max} = \frac{1}{2K\left(C_L\right)_{max\frac{L}{D}}} \tag{3.4.4.4}$$

This was determined in the Annotation to Section 3.4.4.

It is important to note that if a symmetric polar (Raymer Eq. 12.4) is fitted to a set of data for a wing with a cambered airfoil, or for a transport aircraft with a wing set at a positive angle of incidence, and the value of *e* is subsequently calculated, the calculated value will be much higher than if a non-symmetric polar (Raymer Eq.12.5) were fitted. This happens fairly frequently in order to simplify subsequent performance calculations (as occurs in this book). However, the value of *e* so produced is no longer a measure of lifting efficiency. In fact it is quite possible for the value of *e* for a trapezoidal wing to be greater than unity, whereas Prandtl proved that the most efficient wing planform is an ellipse, for which *e* = 1. So it is important to know the basis of the value of *e*. If it is for an asymmetric polar, then the value of $C_{L_{min\,drag}}$ must also be known (see the annotation to section 5.3.7, Fig. 5.3.7.2, for an example of this). It appears that the values of *e* calculated using Raymer Eq. (12.48) and (12.49) are valid for asymmetric polars, and are therefore only valid if the value of $C_{L_{min\,drag}}$ is known. These values should therefore be used with caution.