3.4.4 *L/D* Estimation

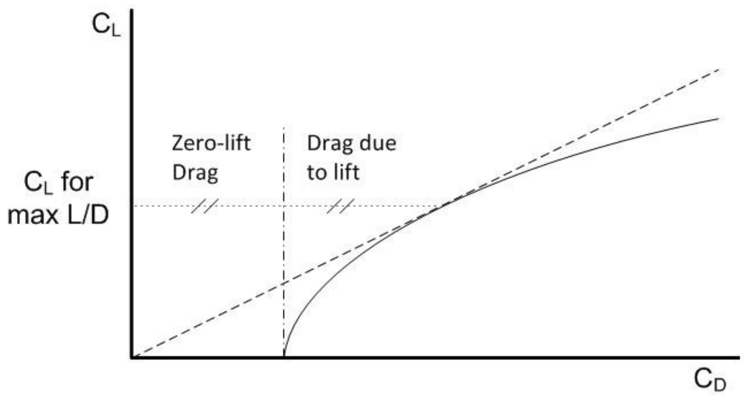

Fig. 3.4.4.1 Calculating C_L at Maximum Lift/Drag Ratio

Ignoring compressibility effects, an aircraft drag polar (Figure 3.4.4.1) may be approximated as:

$$C_D = C_{D_0} + K C_L^2 \tag{3.4.4.1}$$

where K is the "drag due to lift" factor, which will be introduced later in the book, and C_{D_0} is the zero-lift drag.

The point on the polar where C_L/C_D is maximized is where the tangent from the origin touches the curve, i.e., where the gradient of the tangent from the origin equals the gradient of the curve. Thus:

$$\frac{C_D}{C_L} = \frac{dC_D}{dC_L} \tag{3.4.4.2}$$

Differentiating Eq. 3.4.4.1: $$\frac{dC_D}{dC_L} = 2KC_L \tag{3.4.4.3}$$

Thus $$\frac{C_D}{C_L} = 2KC_L \text{ or } C_D = 2KC_L^2 \tag{3.4.4.4}$$

Inserting this back into Eq. 3.4.4.1 we have $C_{D_0} = KC_L^2$, i.e., at the location on the polar where C_L/C_D is maximized, zero-lift drag is equal to drag due to lift.

For the condition of maximum *L/D*: $$C_L = \sqrt{\frac{C_{D_0}}{K}} \tag{3.4.4.5}$$

and $\left(\frac{L}{D}\right)_{max} = \frac{1}{2\sqrt{KC_{D_0}}}$ (3.4.4.6)

This is the condition for optimum endurance (e.g., when loitering), and related to the condition for maximum range, which is usually at a slightly lower C_L for which $\frac{L}{D} = 0.866\left(\frac{L}{D}\right)_{max}$. This latter condition arises from the requirement to maximize *V(L/D)* in the Breguet range equation, which will be discussed in the Annotation to Section 17.2.5.

Raymer's Figure 3.4 shows a classic comparison between two configurations which could have similar mission requirements, but whose design teams took radically different approaches to the design (such as the Boeing B-47 and Avro Vulcan). The delta wing has a much lower zero-lift drag coefficient, even though the values of $(L/D)_{max}$ are about the same. A comparison of drag polars would look something like Fig. 3.4.4.2. The low value of C_{D_o} for the delta wing is not because it has a very low drag, but rather because of the large value of reference wing area that is used in the denominator of the definition of C_D.

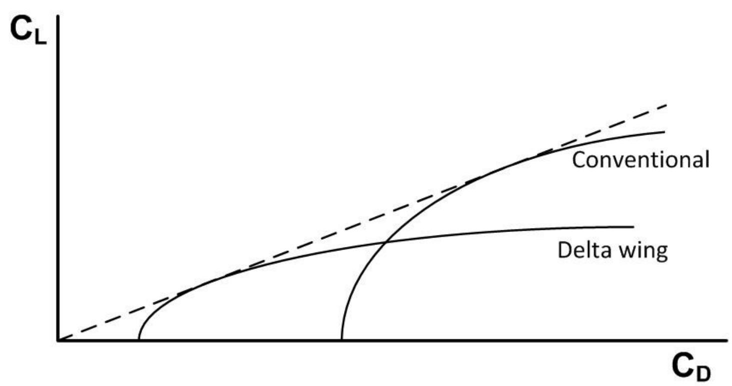

Figure 3.4.4.2 Drag Polar Comparison: Conventional and Delta wing

When flying at $(L/D)_{max}$ (or 0.866 $(L/D)_{max}$) the delta wing is also flying at a lower cruise lift coefficient than the conventional wing, enabling it to fly at a higher Mach number before running into drag divergence. For the Avro Vulcan, the high altitude cruise speed was Mach 0.947 (Ref 3.4.1).

When sizing from a sketch, it is possible to get by without any knowledge of the shape of the drag polar. Raymer Fig. 3.5 shows that you can estimate $(L/D)_{max}$ for a configuration using only a knowledge of the aircraft span and wetted area. You can do this estimation analytically (along with assumptions about skin friction coefficient and Oswald efficiency factor, which will be described in Chapter 17) rather than empirically, and generate results that are close to the lines that represent four of the five classes of airplanes shown in the figure.

If we make the simplifying assumption that the airfoil is uncambered so that the drag polar is symmetric, Raymer Eq. (17.14) shows that

$$C_{L_{mindrag}} = \sqrt{\frac{C_{D_0}}{K}}$$

By definition $C_{D_i} = K C_L^2$ so at this condition $C_{D_i} = K \frac{C_{D_o}}{K} = C_{D_o}$ (as was shown above).

Thus $\left(\frac{L}{D}\right)_{\max} = \left(\frac{C_L}{C_D}\right)_{\max} = \frac{1}{C_D}\sqrt{\frac{C_{D_o}}{K}} = \frac{1}{2\,C_{D_o}}\sqrt{\frac{C_{D_o}}{K}} = \frac{1}{2\sqrt{C_{D_o} K}}$ (3.4.4.7)

K is defined as $K = \frac{1}{\pi A e}$ so $\left(\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{\frac{C_{D_o}}{\pi A e}}}$ (3.4.4.8)

Also, aspect ratio $A = \frac{b^2}{S_{ref}}$ so $\left(\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{\frac{C_{D_o} S_{ref}}{\pi b^2 e}}} = \frac{b}{2\sqrt{\frac{C_{D_0} S_{ref}}{\pi\, e}}}$

(3.4.4.9)

The equivalent skin friction coefficient (defined in Section 12.5) is C_{f_e}, and from Raymer Eq. (12.23)

$$C_{D_o} = C_{f_e} \frac{S_{wet}}{S_{ref}} \qquad (3.4.4.10)$$

Substituting this into Eq. (3.4.4.9) above we arrive at

$$\left(\frac{L}{D}\right)_{\max} = \frac{b}{2\sqrt{\frac{C_{f_e} S_{wet}}{\pi\, e}}} \qquad (3.4.4.11)$$

At this point we have to make some estimates as to the values of C_{f_e} and e. We will assume that $C_{f_e} = 0.0026$ (as shown in Table 12.3) and $e = 0.8$. This will give us the simple result

$$\left(\frac{L}{D}\right)_{\max} = 15.5 \frac{b}{\sqrt{S_{wet}}} \qquad (3.4.4.12)$$

This equation closely matches the line for civil jets in Raymer Fig. 3.6.

For military jets we can assume that C_{fe} is the average of the values for bombers and military transports at 0.00325 (from Table 12.3) and $e = 0.8$, for which we get the approximate result

$$\left(\frac{L}{D}\right)_{\max} = 14 \frac{b}{\sqrt{S_{wet}}} \qquad (3.4.4.13)$$

This equation matches the line for military jets.

For retractable gear propeller-driven aircraft we will assume that $C_{fe} = 0.0048$ and $e = 0.75$, for which we get the result

$$\left(\frac{L}{D}\right)_{\max} = 11 \frac{b}{\sqrt{S_{wet}}} \qquad (3.4.4.14)$$

This equation approximately matches the line for retractable gear propeller-driven aircraft. Similar assumptions can be made to match the line for fixed gear propeller-driven aircraft. Jets at Mach 1.15 have the additional effect of wave drag, so simple assumptions about C_{fe} and e will no longer be applicable.

3.4.4 References

3.4.4.1 Gunston, W., "The Encyclopedia of World Air Power", Crescent Books, 1981