Schaufele Annotations
Chapter 6 Horizontal and Vertical Tail Sizing

Longitudinal Stability and Control

Airplane longitudinal static stability has two requirements:

1. The zero-lift pitching moment (C_{m_o}) must be positive
2. The gradient of pitching moment line (dC_m/dC_L) must be negative

These two requirements will ensure that the pitching moment line intersects the $C_m = 0$ (i.e., the horizontal axis) at a positive value of C_L, and that the airplane is longitudinally statically stable.

The value of C_{m_o} in Fig. 6-1 is set by the airplane geometric characteristics, such as wing camber, flap setting, décalage (the difference in angle of attack between wing and horizontal stabilizer), and elevator deflection. Because of the higher moment arm, décalage and elevator deflection have the strongest effect. The gradient of the pitching moment line (dC_m/dC_L) is defined by the non-dimensional distance of the center of gravity from the neutral point (Eq. 6-1). So there are two simple rules: changes in geometry move the line from left to right, or vice versa (Schaufele Fig. 6-6), and changes in c.g. location change the gradient of the line without changing C_{m_o}.

From Fig. 6-6 it can be seen that when an airplane slows down to land and the lift coefficient increases, the elevator must become increasingly negative (i.e., trailing edge up) to maintain a trimmed condition. In a light airplane the control column may be close to a fully aft position at touchdown.

Schaufele Fig. 6-7 is the primary tool for sizing the horizontal stabilizer. It is also called a "notch" chart. Typically there are two conditions that limit the forward c.g. location:

1. Adequate rotation rate at takeoff rotation with flaps in the takeoff condition
2. Adequate control margin at landing approach with flaps in the landing condition.

This results in two lines marking the forward c.g. limit. The static margin requirement may be different under different flight conditions, resulting in more than one static margin line. In additional the aft c.g. location may be limited by the location of the main landing gear on the wing. In preliminary design the wing is therefore moved forward and aft until the required c.g. range fits as low as possible (i.e., the tail volume is as small as possible) into the notch.

Fig. 6-7 also shows that the locus of the neutral point moves aft as the horizontal tail volume coefficient increases, and continues to do so however large the tail volume coefficient becomes. When the wing and horizontal stabilizer are the same area, the configuration is called a tandem wing, and the neutral point lies just forward of mid-way between the quarter-chord of the m.a.c. of the wing and the quarter-chord of the m.a.c. of the horizontal stabilizer. The tandem wing configuration is aerodynamically rather inefficient so is not often used.

As the rear lifting surface continues to grow, the airplane configuration now becomes a canard, and the neutral point continues to move aft. For a typical canard configuration, the neutral point lies somewhere around the leading edge of the m.a.c. of the aft lifting surface (the wing). The rule for relative location of the c.g. is unchanged, so the c.g. must lie forward of the wing. To minimize c.g. travel, the fuel tanks should be located close to the c.g., which is easy enough to do for a conventional configuration, but for a canard it often requires that the fuel tanks be located on a large glove, or leading edge extension (Fig. 6.1).

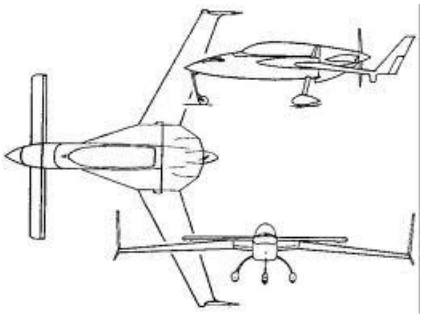

Fig. 6.1 Rutan Varieze

Provided that the two requirements stated at the beginning of this section are met, a canard configuration will be statically stable.

Horizontal Tail Sizing Procedures

Schaufele Fig. 6-8 shows a c.g. range for jet transports of 32% m.a.c. This is valid for a DC-9, with engines at the rear, but is much larger than is necessary for airplanes with engines on the wing.

Airplane type	Forward c.g. limit (%m.a.c.)	Aft c.g. limit (%m.a.c.)	C.g. travel (%m.a.c.)
L-1011	12.0	32.0	20.0
B.707-120	16.0	34.0	18.0
B.720	15.0	31.0	16.0
B.747-200	12.5	32.0	19.5
DC-8-21	16.5	32.0	15.5
A300	11.0	31.0	20.0
DC-9-10	15.0	40.0	25.0
DC-9-33F	3.1	34.7	31.6
B.737-100	15.0	35.0	20.0
F.28	17.0	37.0	20.0
C-130	15.0	35.0	20.0

Source: Torenbeek, et al.

Fig. 6.1 Typical c.g. Travel

Fig. 6.1 shows typical c.g. travel for transport airplanes. The anomaly in the group is the F.28, which has engines on the rear fuselage, but manages to restrict the c.g. travel to that of airplanes with engines on the wing. It is suggested that for transport aircraft with wing-mounted engines, a c.g. travel of 20% m.a.c. be used, and for rear-fuselage-mounted engines a c.g. travel of 25% m.a.c. be used.

Pitch Control

On the DC-9 the elevators and rudder (Fig 6-19 and 6-20) are operated by control tabs in a similar manner to the ailerons (see Annotations to Chapter 4). Additionally, the elevators have a hydraulic boost when the control column is in the full forward position to provide extra control power to get out of a deep stall. Control columns are linked in pitch by a frangible torque tube. The port elevator control tab is controlled by the captain's control column and the starboard control tab by the first officer's control column. Failure of the pitch control system on one side can be overridden by breaking the torque tube. Fig 6-19 shows two tabs on each side. This was done because a single tab of the required span would have been structurally unstable. The tabs on each side operate together.

On commercial aircraft, pitch trim usually accomplished by moving the horizontal stabilizer. On the DC-9 a single jack screw in the vertical stabilizer is used to move the horizontal stabilizer.

The use of control tabs is unusual for a commercial airplane but not unique; the BAC 1-11 used a similar system.

Horizontal and Vertical Stabilizer Geometric Characteristics

It is important that an aircraft experience a nose-down pitching moment at the stall, which implies that for a conventional layout the wing must stall before the horizontal stabilizer, and for a canard layout, the canard stall before the wing. One way to ensure this is for the conventional horizontal stabilizer to have a higher sweep and lower aspect ratio than the wing. Schaufele’s suggestion that horizontal stabilizer have a 5^0 higher sweep than the wing is reasonable. Conversely, a canard surface should have a higher aspect ratio and lower sweep than the wing, as illustrated by the Rutan Varieze in Fig. 6.1 above.

If an airplane has a T-tail, the vertical stabilizer must carry heavy loads. For this reason the taper ratio is very large (i.e., it has a small, or zero, taper) as illustrated in Schaufele Fig. 6-20. If the horizontal stabilizer is mounted on the rear fuselage then the vertical stabilizer has a taper ratio which is comparable with the horizontal stabilizer.

Calculating V_{MC}

(This section is adapted from Schaufele's class handout).

The minimum speed for which it is possible to maintain directional equilibrium ($C_n = 0$) during takeoff roll with one engine inoperative is called the minimum control speed, V_{MC}. It may be obtained graphically by finding the intersection of the yawing moment due to the inoperative engine and the yawing moment due to full directional control, as shown in Schaufele Fig. 6-14. The procedure is as follows:

<u>Calculate the yawing moment due to the most critical inoperative engine</u>

- Use the data from Fig. 8-5 on T/T_{static} for your bypass ratio
- Multiply the value by the SLST of one engine
- Multiply by the moment arm of the engine thrust line from the c.g. to obtain the data for a chart of yawing moment (N_{oei}) vs. velocity with one engine inoperative.

Calculate the yawing moment available from full rudder deflection at zero sideslip vs. velocity

The following is taken from Schaufele's handout (simplified version)

- Consider the vertical tail as one half of a low aspect ratio wing at zero angle of attack, with the rudder as a trailing edge flap
- Consider the rudder as a plain trailing edge flap. For a flap deflection equal to the maximum rudder deflection (based on Fig. 6-13, assume 30^0), use the method in Schaufele page 220 to calculate the change in zero lift angle for that rudder deflection ($\Delta\alpha_{OL}$). This angle will be negative, as illustrated in Fig 11-3
- From Schaufele Fig. 11-1, calculate the value of the lift curve slope, C_{L_V}, treating the vertical stabilizer as a wing on its side. Factor this value by 1.5 to account for the end plate effect of the horizontal stabilizer
- For this value of C_{L_V}, plot the lift curve slope, starting from the value of α_{OL}
- Calculate the value of C_{L_V} at a sideslip angle of zero (i.e., $\alpha = 0$, using wing nomenclature)
- Using the distance from the aftmost c.g. location to the quarter-chord of the m.a.c. of the vertical stabilizer (l_V) as the moment arm, calculate the aerodynamic moment, N, at several values of airspeed from $N = C_{L_V} \, l_V \, q \, S_{VT}$

Plot calculated values

Plot the values of engine-out yawing moment and aerodynamic moment due to full rudder deflection, as illustrated in Schaufele Fig. 6-14. The intersection of the two lines determines the value of V_{MC}.

Design Exercise

For conceptual design, you only need to get a reasonable estimate of the horizontal and vertical tail geometry. Note that throughout Chapter 6 Schaufele uses the term "tail volume" (which has a dimension of length^3), when he means "tail volume coefficient" (which is dimensionless).

For the horizontal stabilizer, use Fig. 6-9 to estimate the horizontal tail volume coefficient per unit c.g. range. Then use Fig. 6-8 to estimate the c.g. range (unfortunately you don't actually calculate the c.g. range until Chapter 10), and hence obtain the horizontal tail volume coefficient. In Fig. 6-8 the value of c.g. range for jet transports is much too high. See Figure 6.1 above for more typical values. The value of the horizontal tail volume coefficient should be within the range shown in Fig. 6-10. Use Fig. 6-17 for other characteristics of the horizontal stabilizer. Measure the characteristics of airplanes that are similar to the one you are designing, and pick similar characteristics. The aspect ratio of the horizontal stabilizer must be much less than that of the wing, so that the wing will stall before the horizontal stabilizer.

For the vertical stabilizer, use Fig. 6-15 to estimate the value of the vertical tail volume coefficient. Use Fig. 6-18 to select other tail characteristics. You can find more data for both horizontal and vertical stabilizers in Roskam Volume III, Chapter 6.

The foregoing procedures for estimating volume coefficients are empirical. In preliminary design, the horizontal tail volume coefficient is calculated using the method described on Schaufele page 145. It requires fairly detailed knowledge of the geometry of the rest of the airplane, plus takeoff and landing speeds. Similarly, the vertical stabilizer is sized using a variety of criteria, including the requirement that $V_2 > 1.1V_{MC}$ (Fig. 6-14), where V_2 is the takeoff safety speed ($V_2 > V_{S1}$ where V_{S1} is the stall speed in the takeoff condition with gear up) and V_{MC} is the minimum control speed with the critical engine inoperative. The foregoing is an approximation to enable initial vertical stabilizer sizing. The topic of takeoff speeds will be discussed in Chapter 16.