Chapter 12
Aerodynamic Analysis

Breguet Range Equation Design Drivers

Louis Charles Breguet pilotant son premier aéroplane (Breguet I) en juin 1909 à La Brayelle près de Douai. (Musée de l'Air)

Source: Musée de l'Air

For given speed of sound, a, and initial weight, $W_{initial}$

$$R = \frac{a}{c}\left[M\left(\frac{L}{D}\right)\right]\ln\left(\frac{W_{initial}}{W_{final}}\right)$$

Propulsion

Aerodynamics

Structures and Materials

Drag Polar

"Induced drag coefficient" is a misnomer" because C_{D_i} <u>includes</u> viscous drag due to lift

$$C_D = C_{D_0} + C_{D_i}$$

$$C_D = C_{D_0} + \frac{1}{\pi AR\, e} C_L^2$$

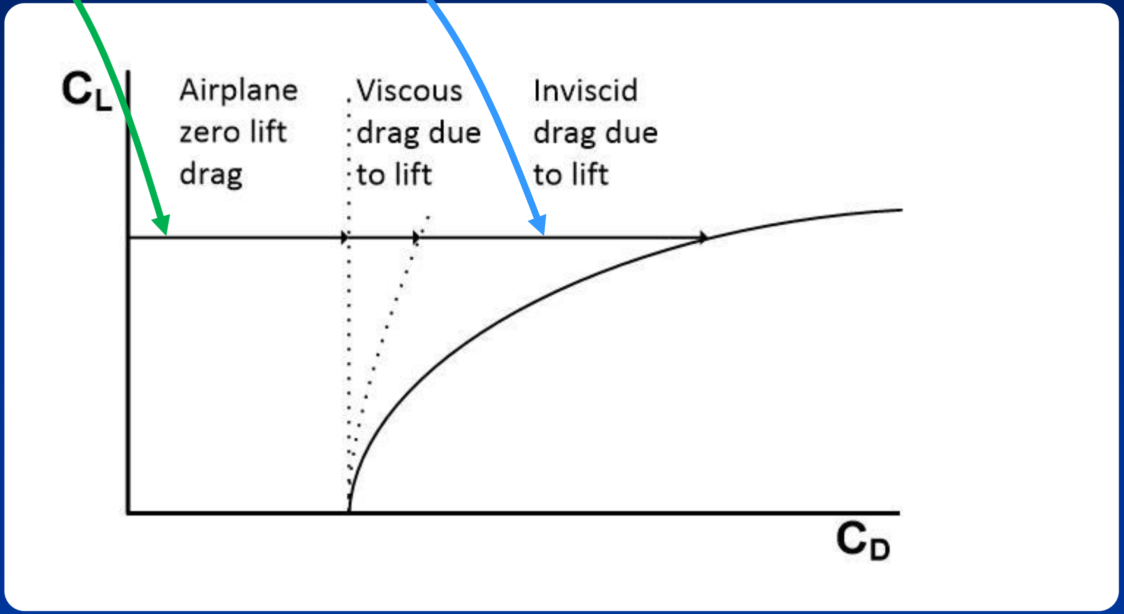
$$C_D = C_{D_0} + K C_L^2$$

where

e = Oswald efficiency factor

K = Drag-due-to-lift factor

Need these two values



Drag polars are often assumed to be symmetrical to simplify analysis. In reality, for most aircraft, they are not

Topics in Raymer Chapter 12

	Subsonic	Transonic	Supersonic
C_L vs α	12.4.1	12.4 Mach correction	12.4.2
C_{Lmax} (clean)	12.4.5		12.4.5
C_{Lmax} (high lift devices)	12.4.6		12.4.6
Parasite Drag	12.5		12.5.9 Area Rule
Drag due to lift	12.6.1 Oswald Span Efficiency	12.5.10 M_{DD} (drag divergence)	12.6.2 Leading Edge Suction

Lift and High Lift Systems: C_L vs α, C_{Lmax} (clean), C_{Lmax} (high lift devices)

Zero-Lift Drag → Parasite Drag

Drag due to lift → Drag due to lift

This topic also addressed in Section 4.3

Lift and High Lift Systems

Zero-Lift Drag C_{D_0}

Drag due to Lift C_{D_i}

Wave Drag due to Volume $C_{D_{0_{supersonic}}}$

Wave Drag due to Lift C_{D_W}

Wing Design

Lift

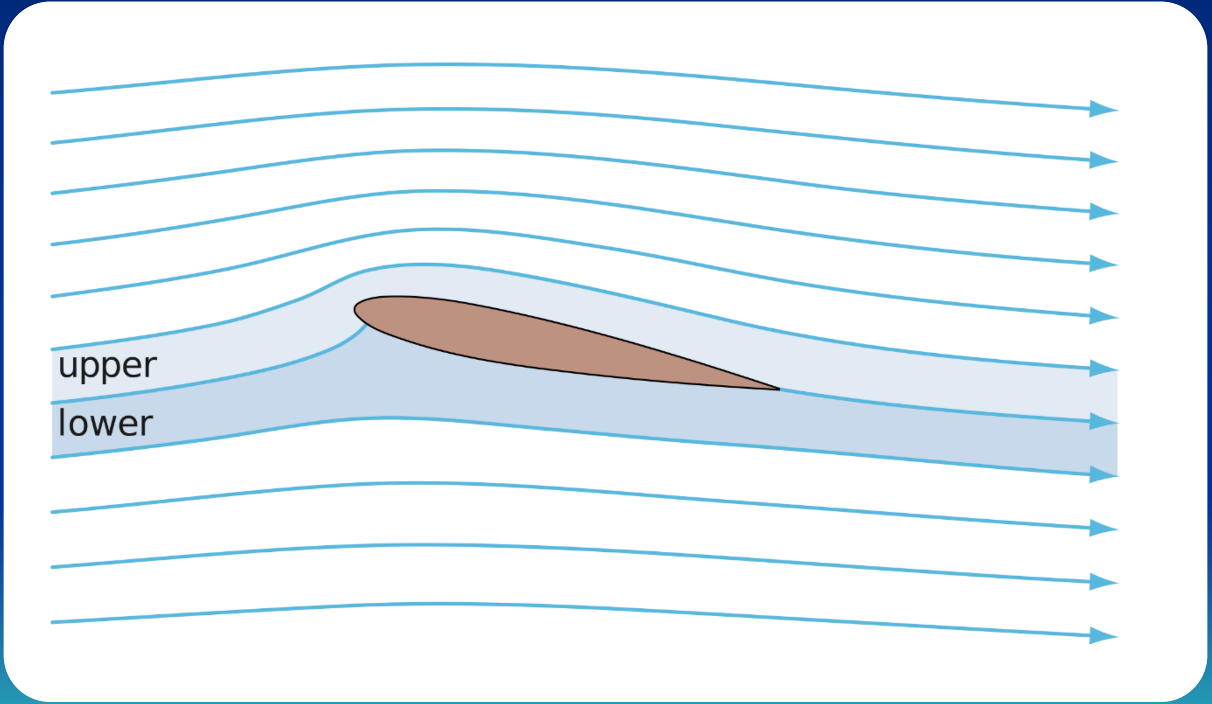
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho \cancel{g h_1} = P_2 + \frac{1}{2}\rho v_2^2 + \rho \cancel{g h_2}$$

From Bernoulli's equation

Flow accelerates over upper surface so air pressure is lower

Stagnation streamline

Flow slows down over lower surface so air pressure is higher



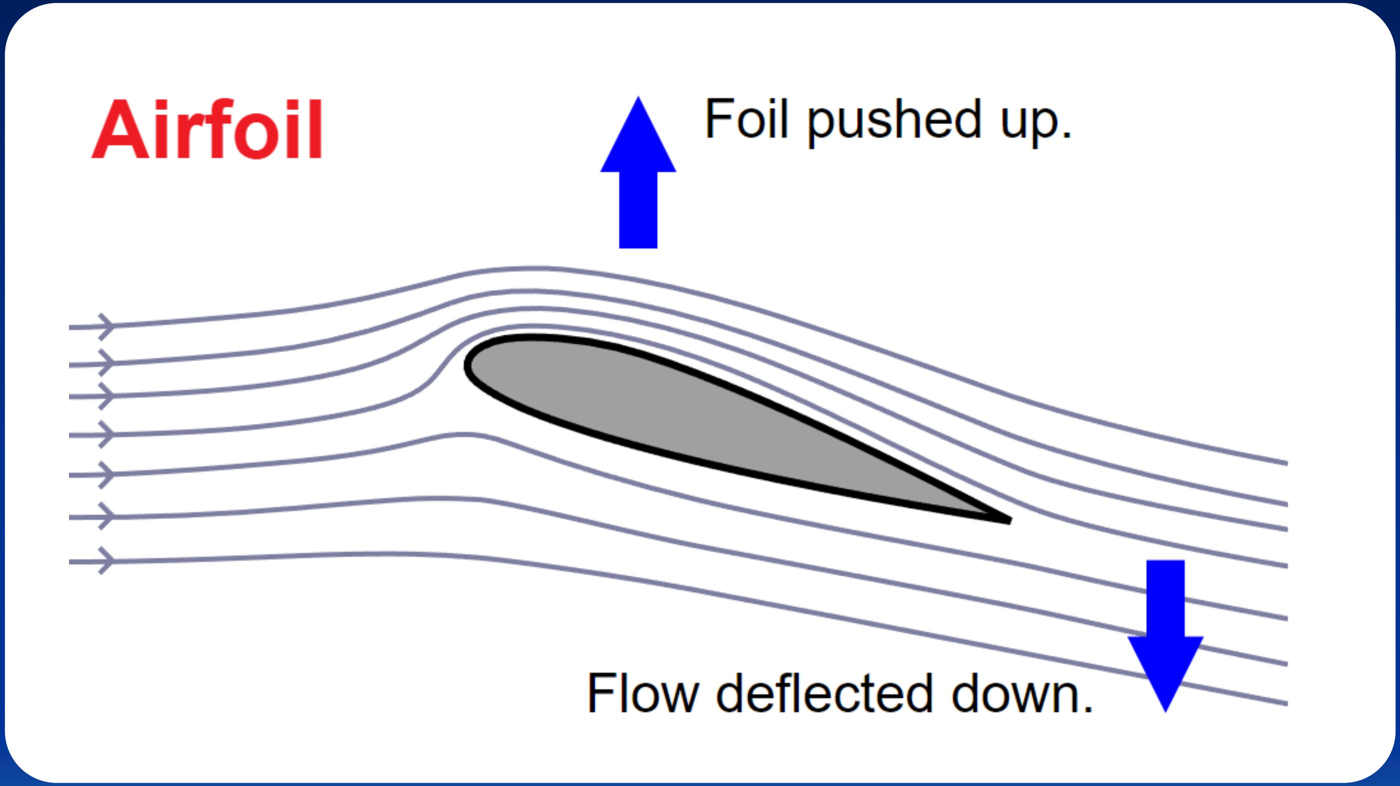
https://en.wikipedia.org/wiki/Lift_(force)

Lift

If you think you understand aerodynamics, then you probably don’t

Read Doug McLean: “Understanding Aerodynamics: Arguing from the Real Physics” Wiley, 2013

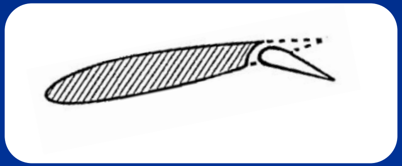
The laws of aerodynamics are mathematical models, not physical models

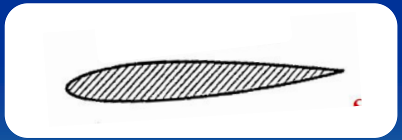


https://en.wikipedia.org/wiki/Lift_(force)

Section C_l vs. α plot

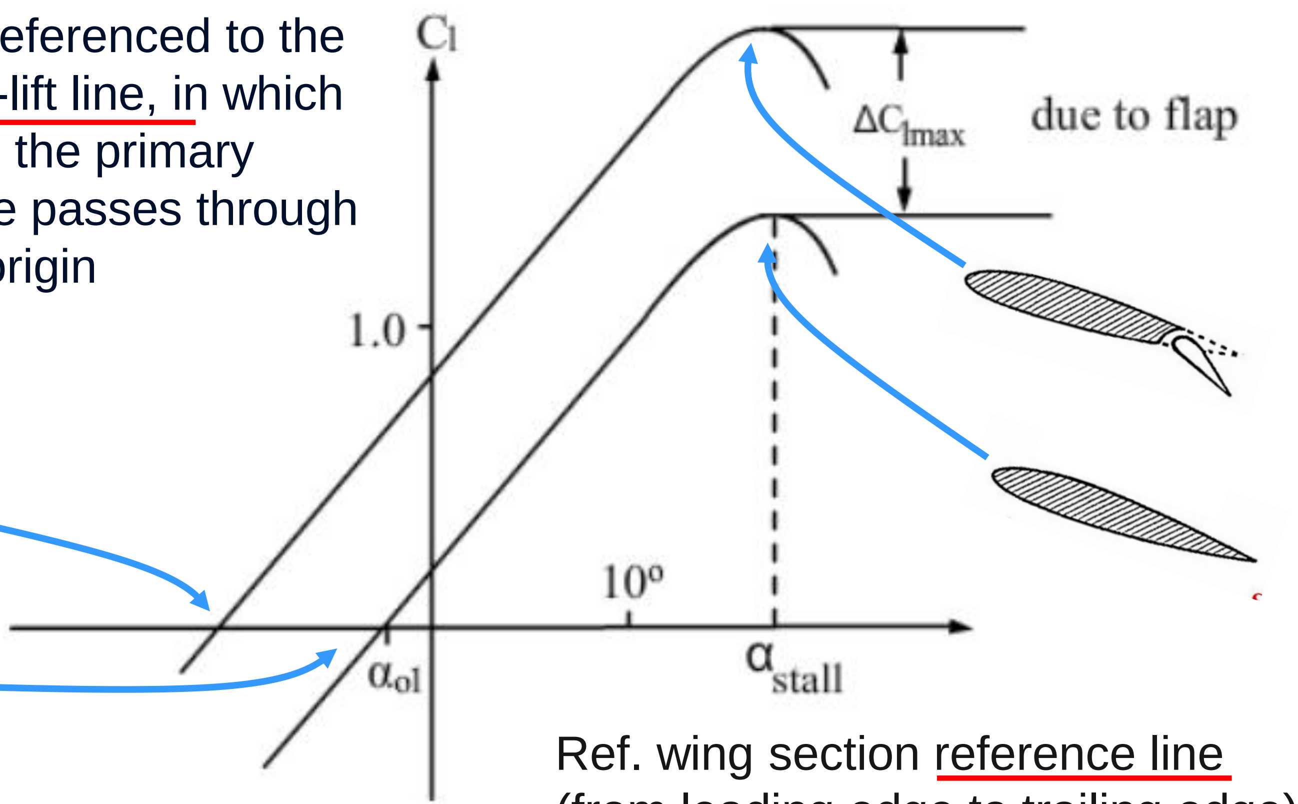
Cambered airfoil section with slotted flap





α_{ol} is negative

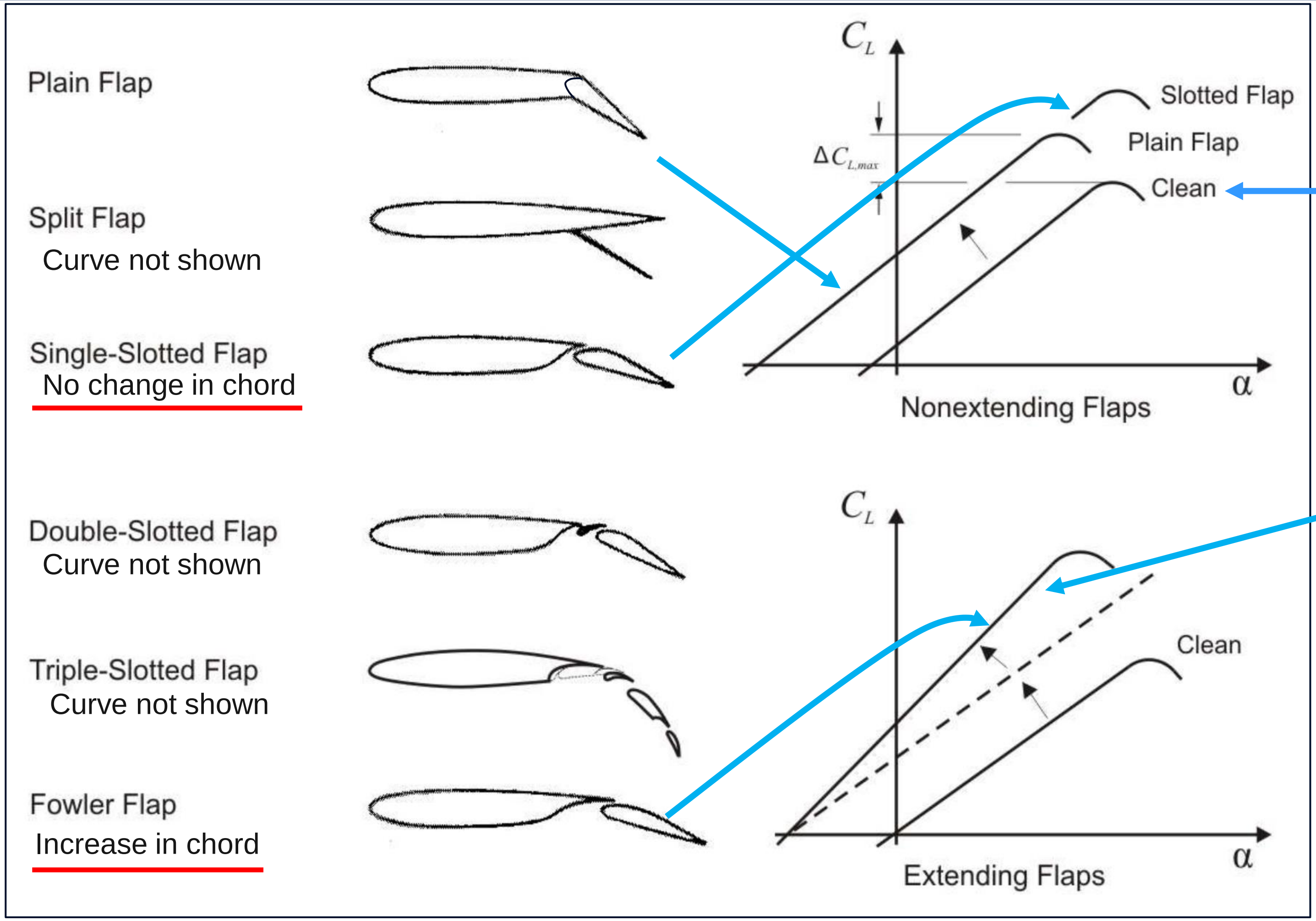
Sometimes curves are referenced to the zero-lift line, in which case the primary curve passes through the origin



Ref. wing section reference line (from leading edge to trailing edge)

https://en.wikipedia.org/wiki/Lift_(force)

Trailing Edge Flap Systems



i.e. flaps not deployed

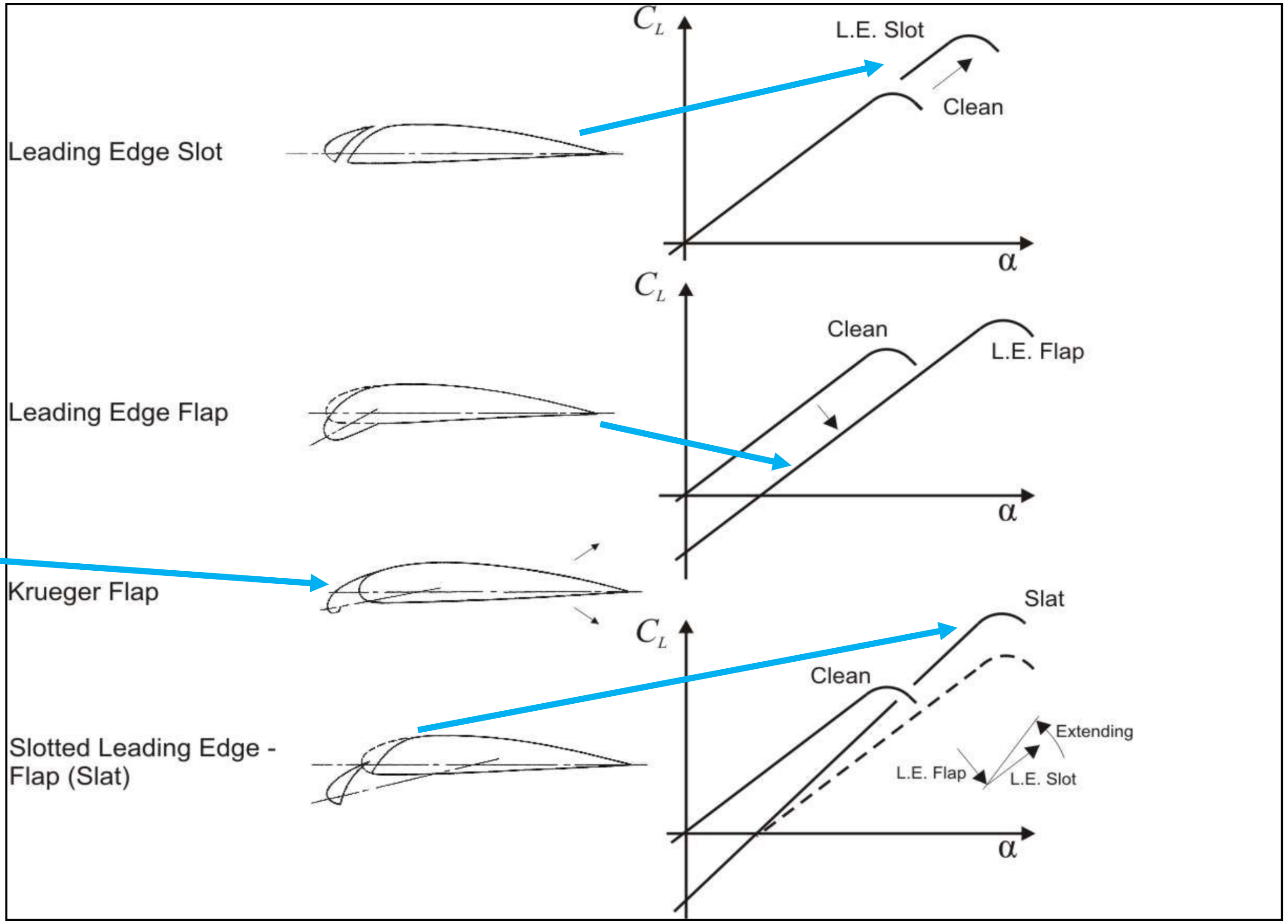
Increase in gradient of curve because reference chord in definition of C_L is original wing chord, not extended chord

In reality, nearly all slotted flaps have Fowler action

https://www.fzt.haw-hamburg.de/pers/Scholz/HOOU/AircraftDesign_8_HighLift.pdf

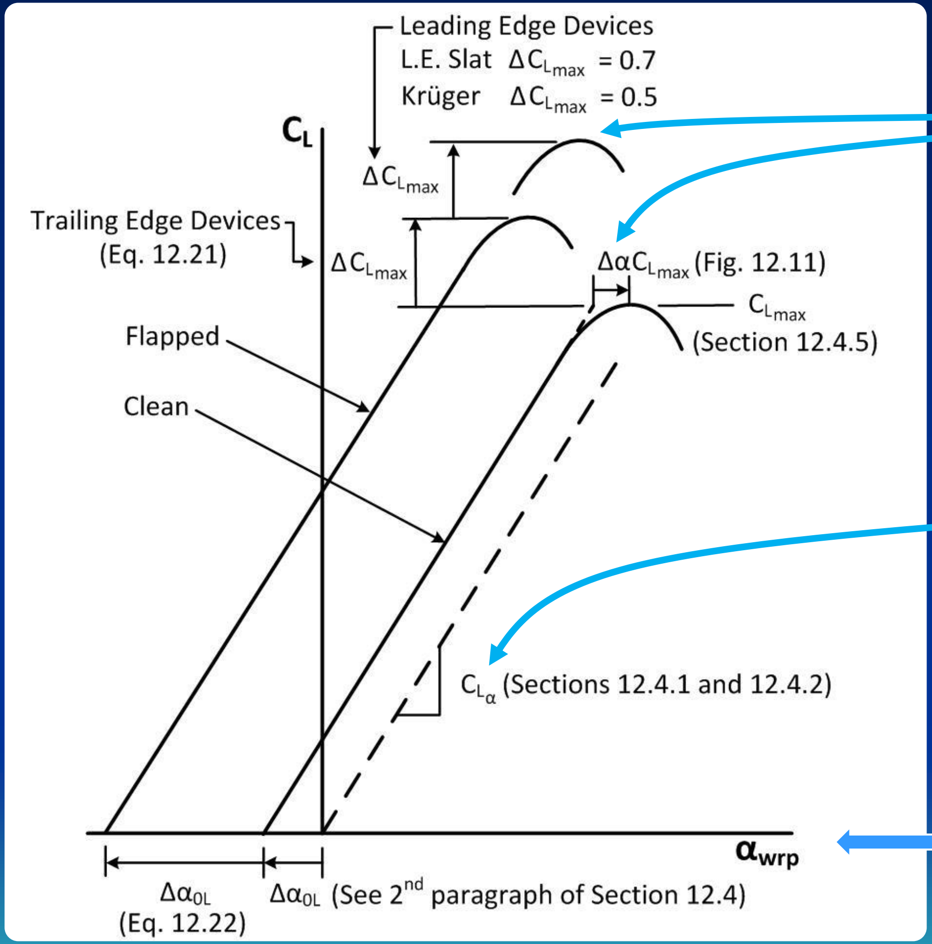
Leading Edge Flap/Slat Systems

Krueger flap either translates or rotates



https://www.fzt.haw-hamburg.de/pers/Scholz/HOOU/AircraftDesign_8_HighLift.pdf

Generation of C_L vs. α Plot



Use for

- Landing gear length
- Cockpit visibility
- Takeoff and landing speeds

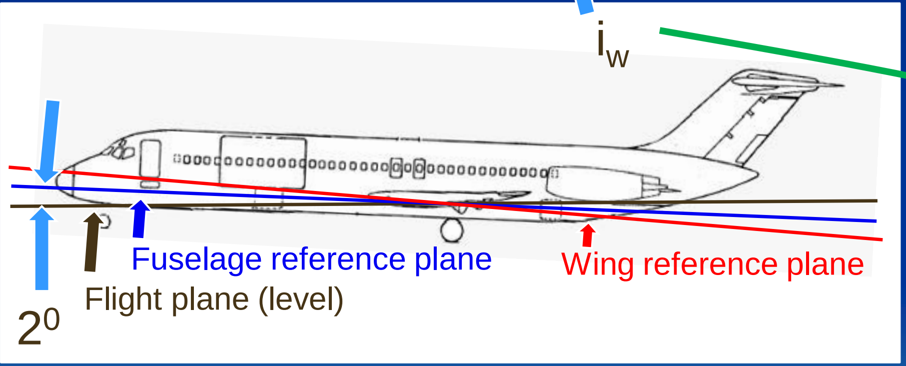
Use for

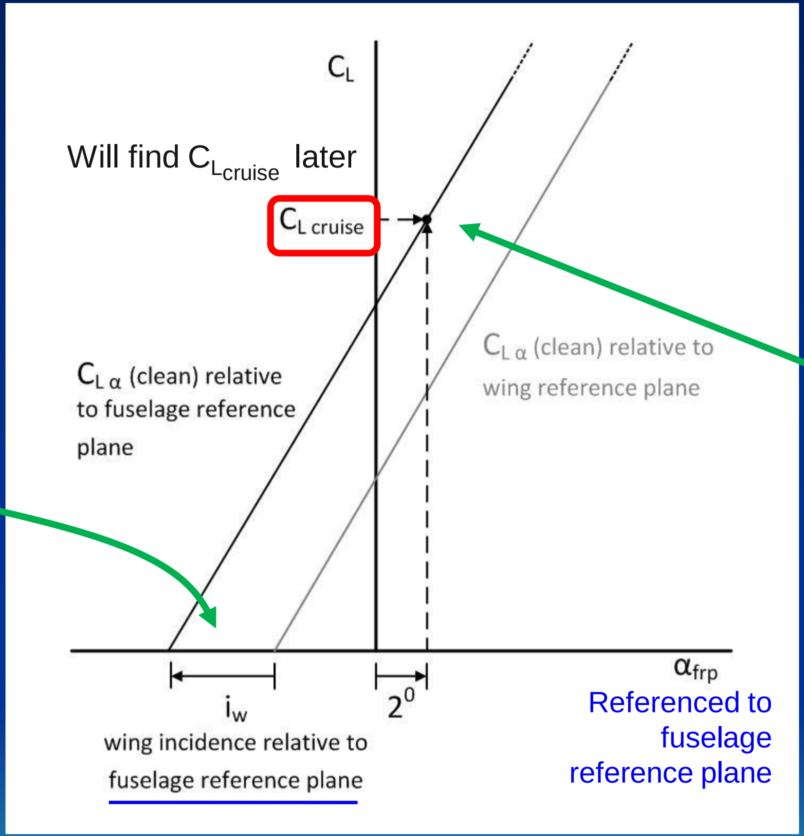
- setting wing reference plane relative to fuselage reference plane
- S&C analysis (e.g Raymer Eq. 16.9)

Values here are plotted wrt. wing reference plane

Translating C_L vs. α Plot to FRP

Set wing on fuselage for fuselage attitude of 2^0 at typical cruise C_L





Move the C_L vs. α curve so that it passes through this point

For a given C_L, $\alpha_{FRP} < \alpha_{WRP}$

C_L vs. α Gradient

Λ_{maxt} is sweep (in rad) of sweep of max. thickness chord

Fuselage correction

NACA airfoil data in N&C, Appendix F.2

Raymer Eq. 12.6

$$C_{L_\alpha} = \frac{2\pi A}{2 + \sqrt{4 + \frac{A^2\beta^2}{\eta^2}\left(1 + \frac{\tan^2 \Lambda_{maxt}}{\beta^2}\right)}} \boxed{\left(\frac{S_{exposed}}{S_{ref}}\right)(F)}$$

Raymer Eq. 12.7

where $$\beta^2 = 1 - M^2$$

Raymer Eq. 12.8

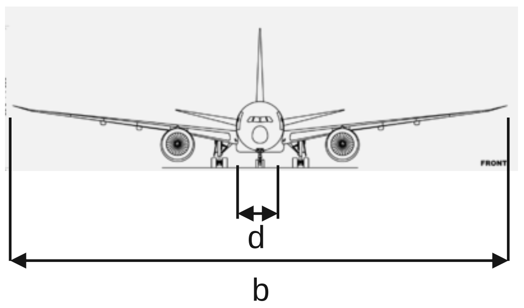
$$\eta = \frac{C_{l_\alpha}}{2\pi / \beta}$$

For low M, $\beta^2 \approx 1$

In theory, $C_{l_\alpha} = 2\pi$ so for low M this term reduces to A^2

Raymer Eq. 12.9

$$F = 1.07\left(1 + \frac{d}{b}\right)^2$$



Low Speed C_L vs. α Gradient

Raymer Eq. 12.6

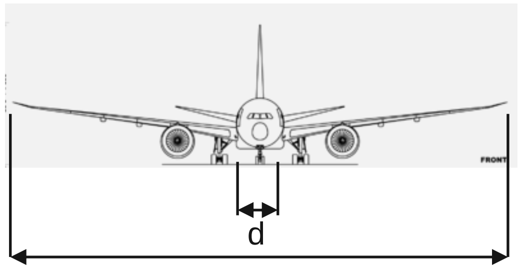
$$C_{L_\alpha} = \frac{2\pi A}{2 + \sqrt{4 + A^2\left(1 + \tan^2 \Lambda_{max\,t}\right)}} \boxed{\left(\frac{S_{exposed}}{S_{ref}}\right)(F)}$$

Fuselage correction (if > 1 then set to 0.98)

Raymer Eq. 12.9

$$\text{where } F = 1.07\left(1 + \frac{d}{b}\right)^2$$

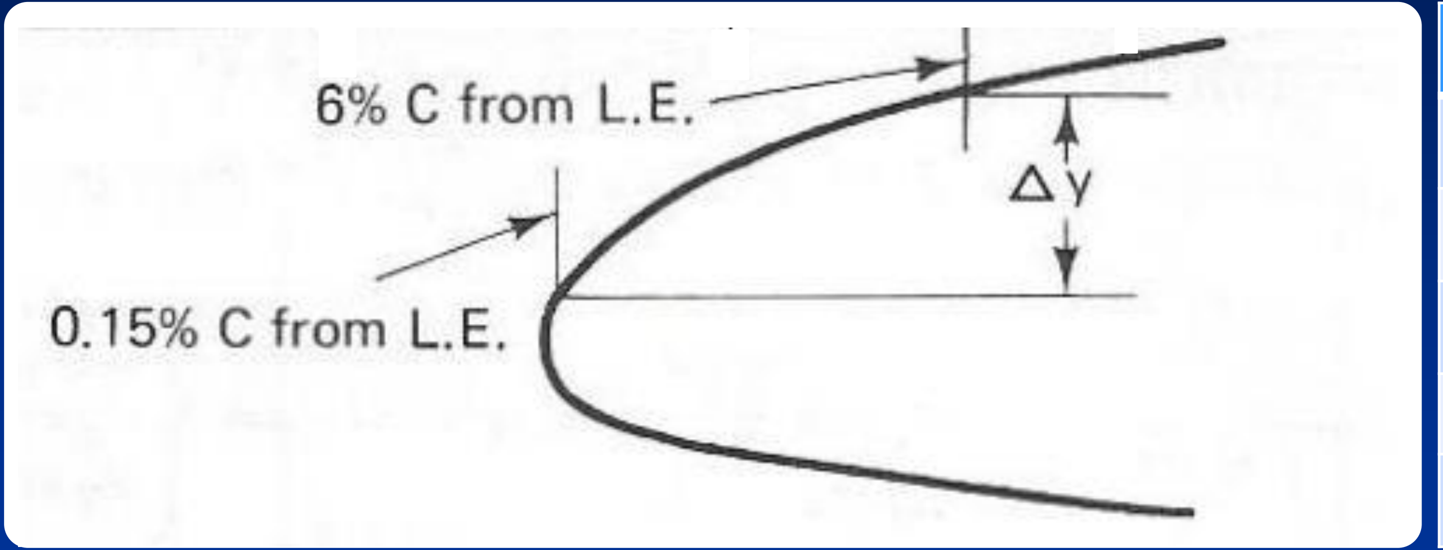
Λ_{maxt} is sweep (in rad) of location of max. thickness



Wing Max Lift Coefficient

$$C_{L_{max}} = (C_{L_{max}})_{clean} + (\Delta C_{L_{max}})_{flaps+slats}$$

Δy For Common Airfoils



Airfoil Type	Δy (%)
NACA 4 digit	26 t/c
NACA 5 digit	26 t/c
NACA 64 series	21.3 t/c
NACA 65 series	19.3 t/c
Biconvex	11.6 t/c

Separation likely to occur near L.E.

Raymer Table 12.1

Typically t/c = 0.1 so for NACA 65 series $\Delta y \approx 2\%$

Also see Nicolai & Carichner (Vol 1) Fig 9.17

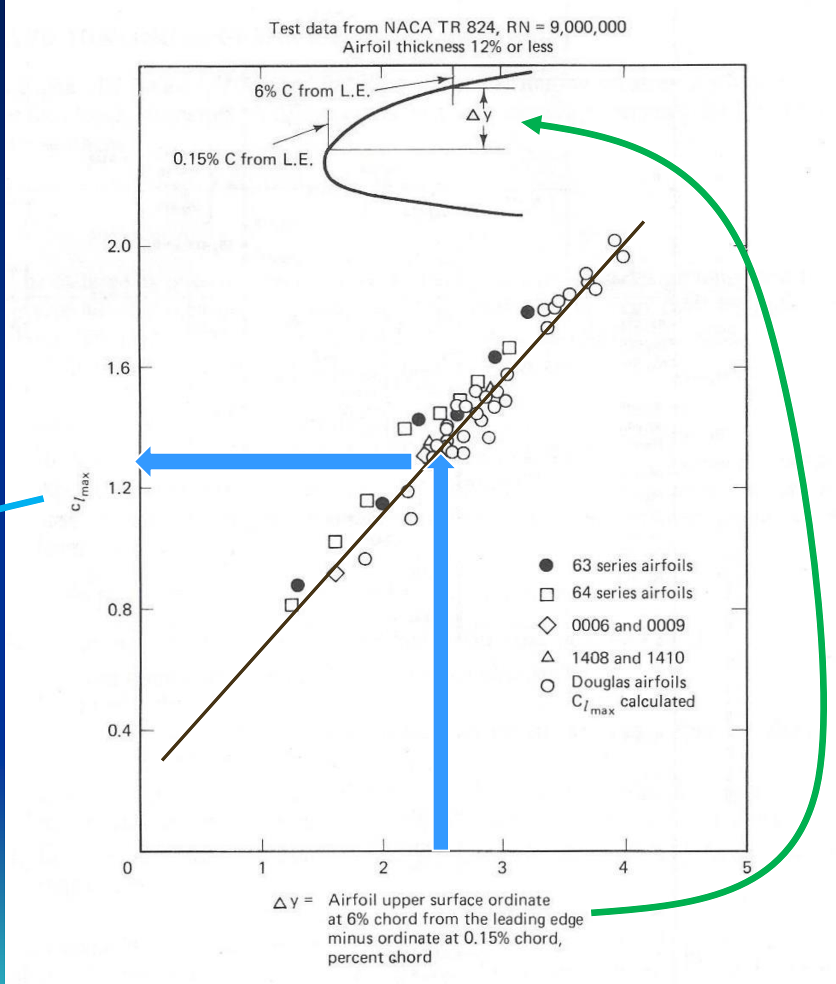
Estimation of Clean $C_{L_{max}}$ with Known Airfoil Section

Assume that Mach correction is included in 0.9 value

For high AR wing with moderate sweep and $\frac{t}{c} \leq 12\%$

$$\boxed{C_{L_{\max}} = 0.9\, C_{l_{\max}} \cos\Lambda_{0,25c}}$$

Raymer Eq. (12.15)



Shevell Fig. 14.1

E.g. $\Delta y = 2.5\%$

So $C_{l_{max}} = 1.3$

For $\Lambda_{c/4} = 32^o$

$C_{L_{max}}/C_{l_{max}} =$

$0.9 \times 1.3 \times 0.848$

$C_{L_{max}} = 0.99$

Assume

AR=8, λ=0.25

Estimation of $C_{L_{max}}$ for t/c > 12%

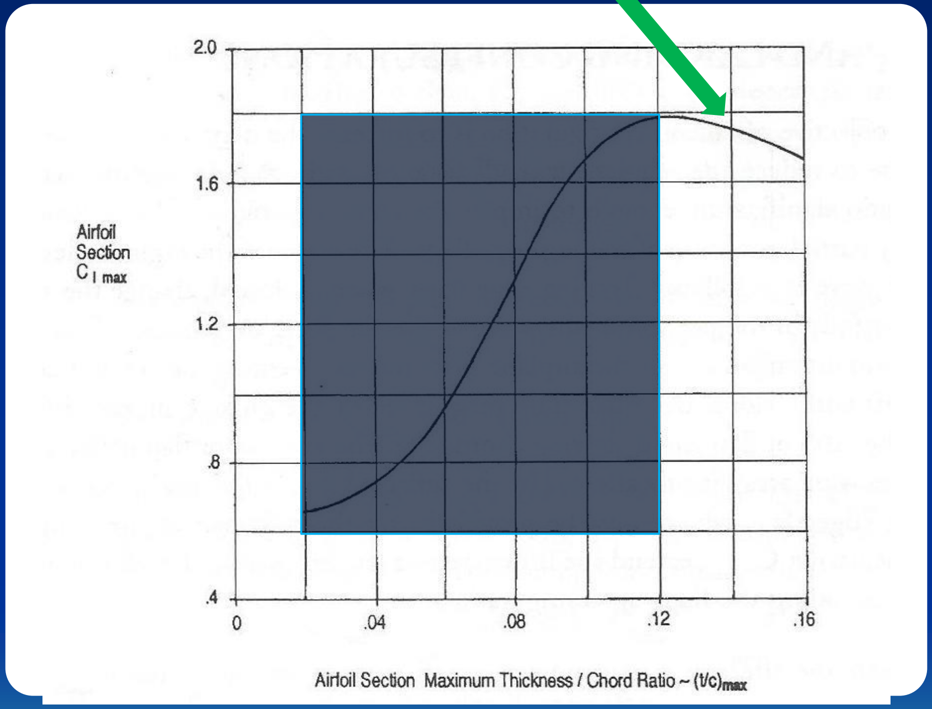
For high AR wing

with moderate sweep

$$C_{L_{max}} = 0.9\, C_{l_{max}} \cos\Lambda_{0,25c}$$

Raymer Eq. (12.15)

For wing with t/c > 12%

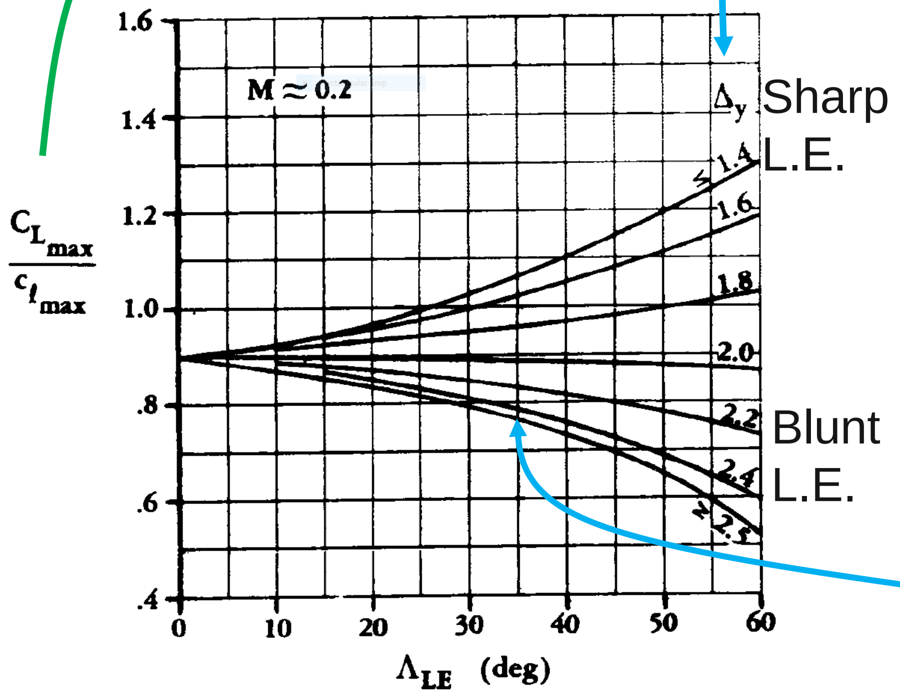


From: Schaufele Fig. 11-4

If t/c > 12%, then initial separation is more likely to occur aft of midchord (probably doesn't apply to supercritical airfoils)

Estimation of Clean C_{Lmax}

Sharp L.E. generates strong streamwise vortices



Raymer Fig. 12.10 Correction for M as fn. Λ_{LE}
For takeoff and landing $\Delta C_{Lmax} \approx -0.03$

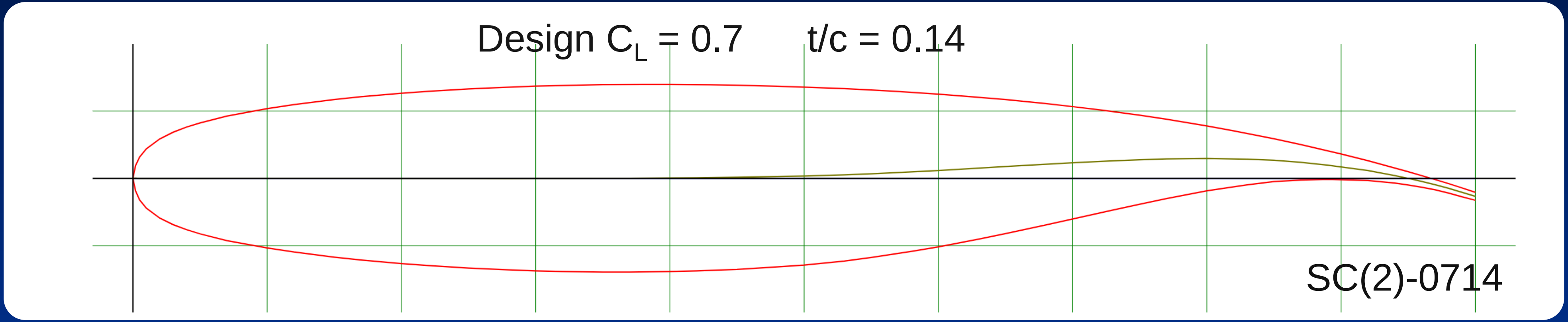
For high AR wing

$$C_{L_{max}} = C_{l_{max}} \left(\frac{C_{L_{max}}}{C_{l_{max}}} \right) + \Delta C_{L_{max}}$$

E.g. $\Delta y = 2.5\%$
so $C_{lmax} = 1.3$ (Shevell)
On Raymer Fig.12.9
For $\Lambda_{LE} = 35^o$
$C_{Lmax}/C_{lmax} = 0.8$
$C_{Lmax} = 1.3 \times 0.75 - 0.03 = 0.945$

For C_{lmax} data, see Raymer Appendix D, for Abbott & von Doenoff data, or Nicolai & Carichner, Appendix F

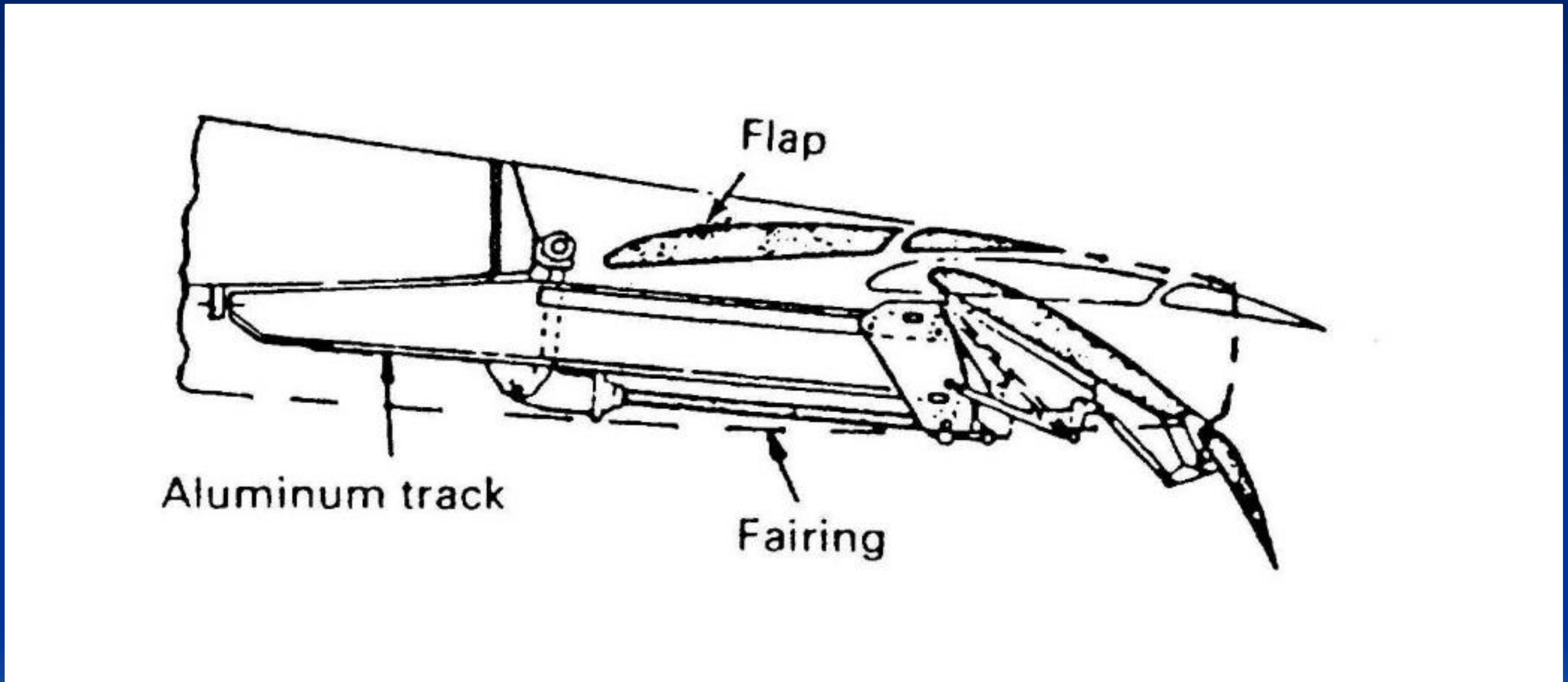
Supercritical Airfoil Sections



Most modern commercial aircraft have proprietary wing sections

For conceptual designer, accept what drag polars and C_L vs. α data the aerodynamics group gives you!

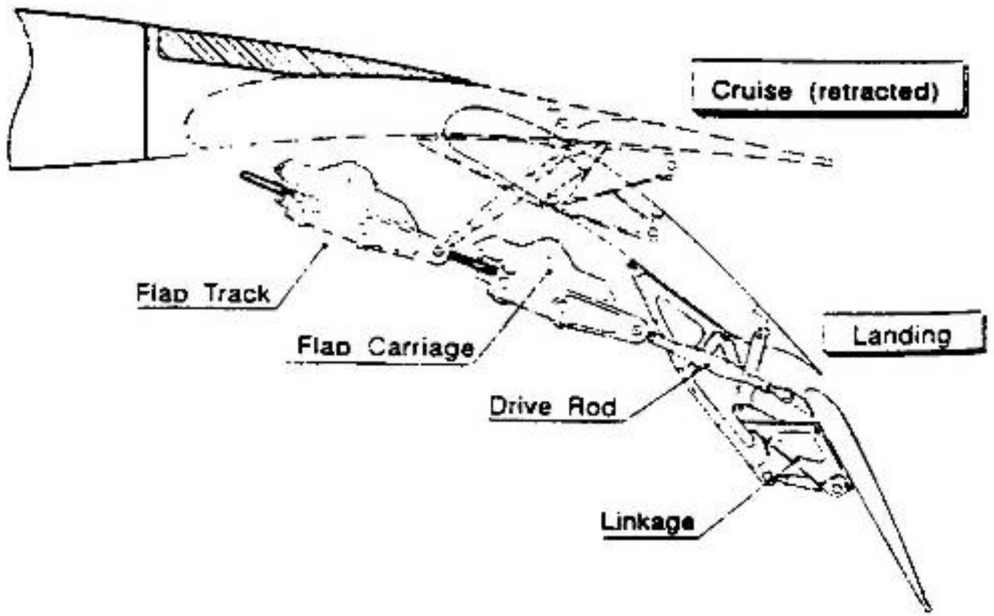
A300B Flap System



- Double-slotted
- Extends on flap tracks

ADAC
Aircraft Design & Consulting

A321 Flap System

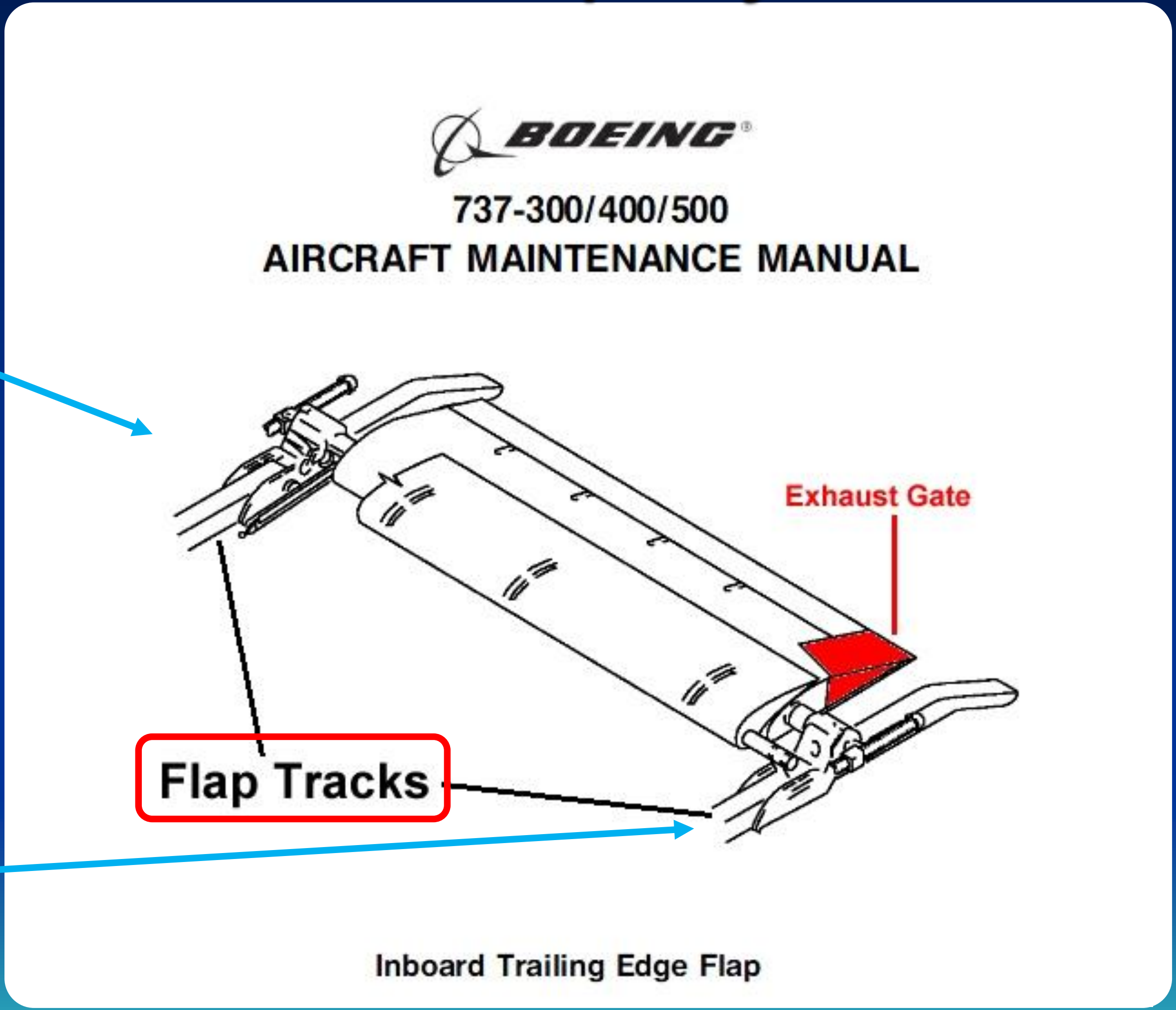


737 Flap System

In wing root fairing

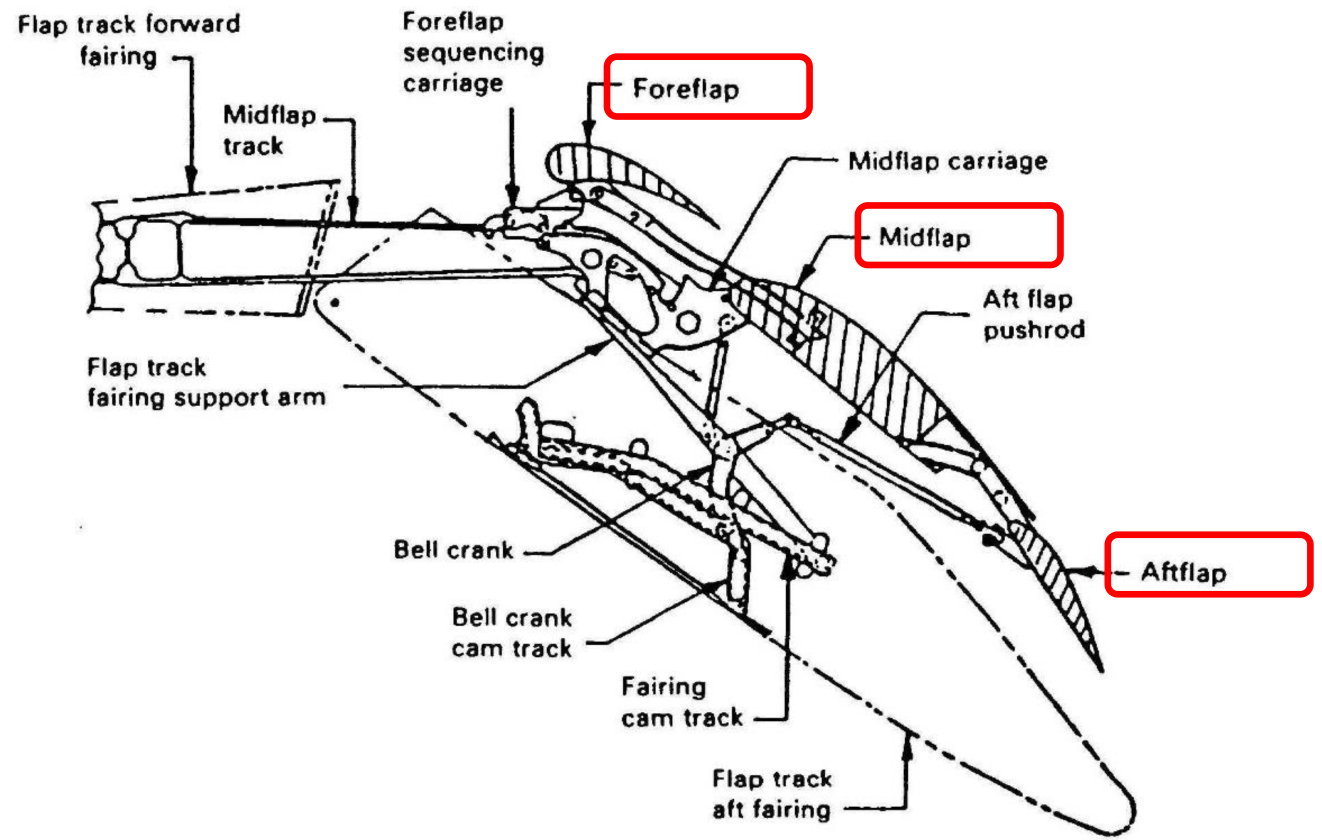
- Triple-slotted
- Extends on flap tracks

In engine aft fairing



737 Mid-flap System

- Triple-slotted
- Extends on flap tracks
- Tracks on either end of flap not shown here



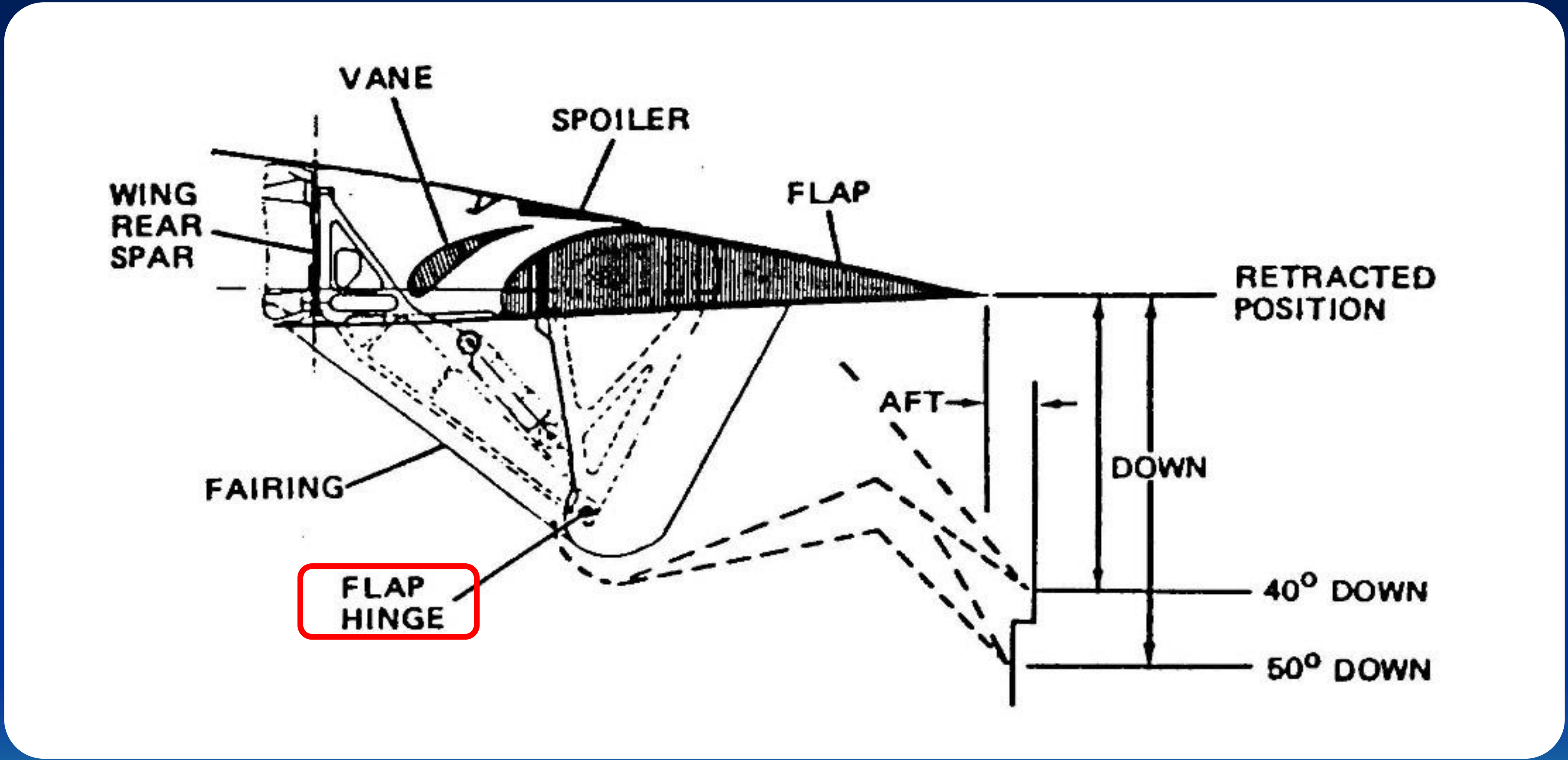
Flap Track Canoes

- Tips painted red to avoid damage

© Taha Ashoori on Airliners.net

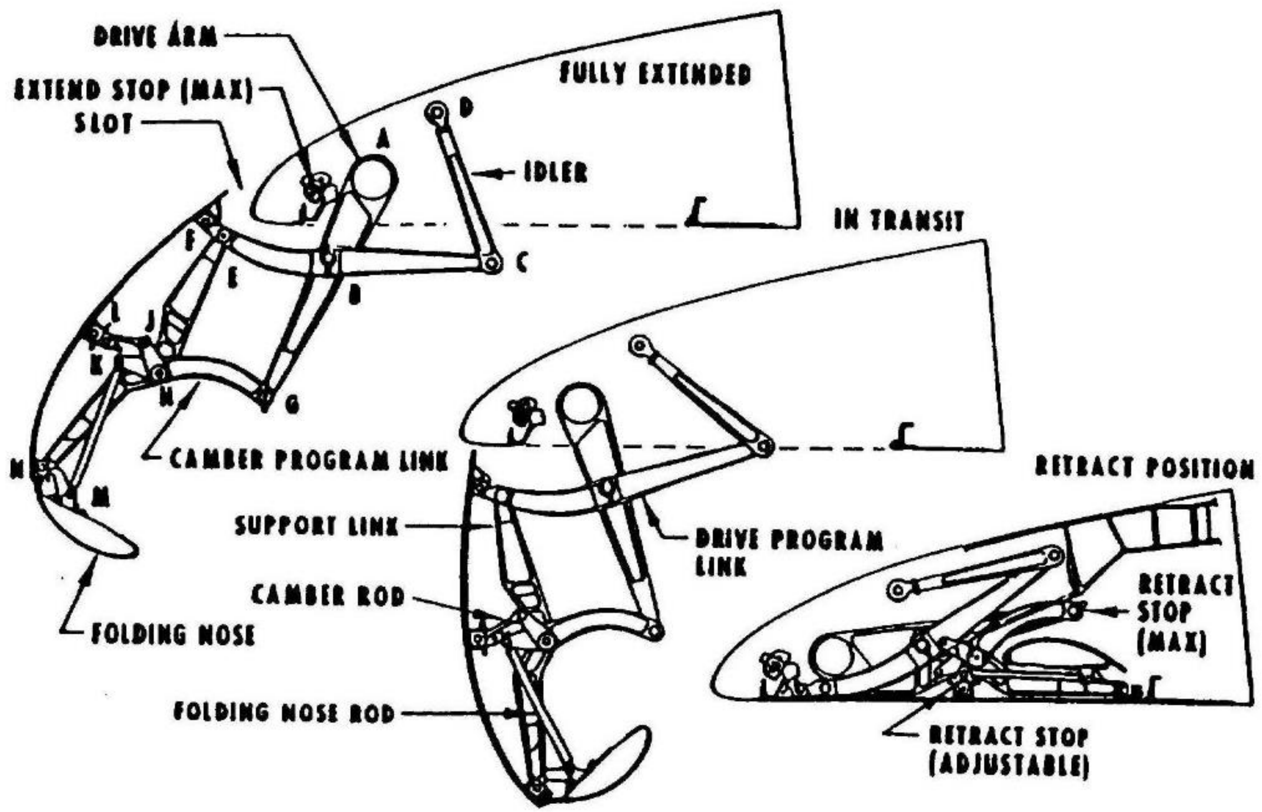
DC-9 Flap System

Limited in choice of flap angle vs. extension



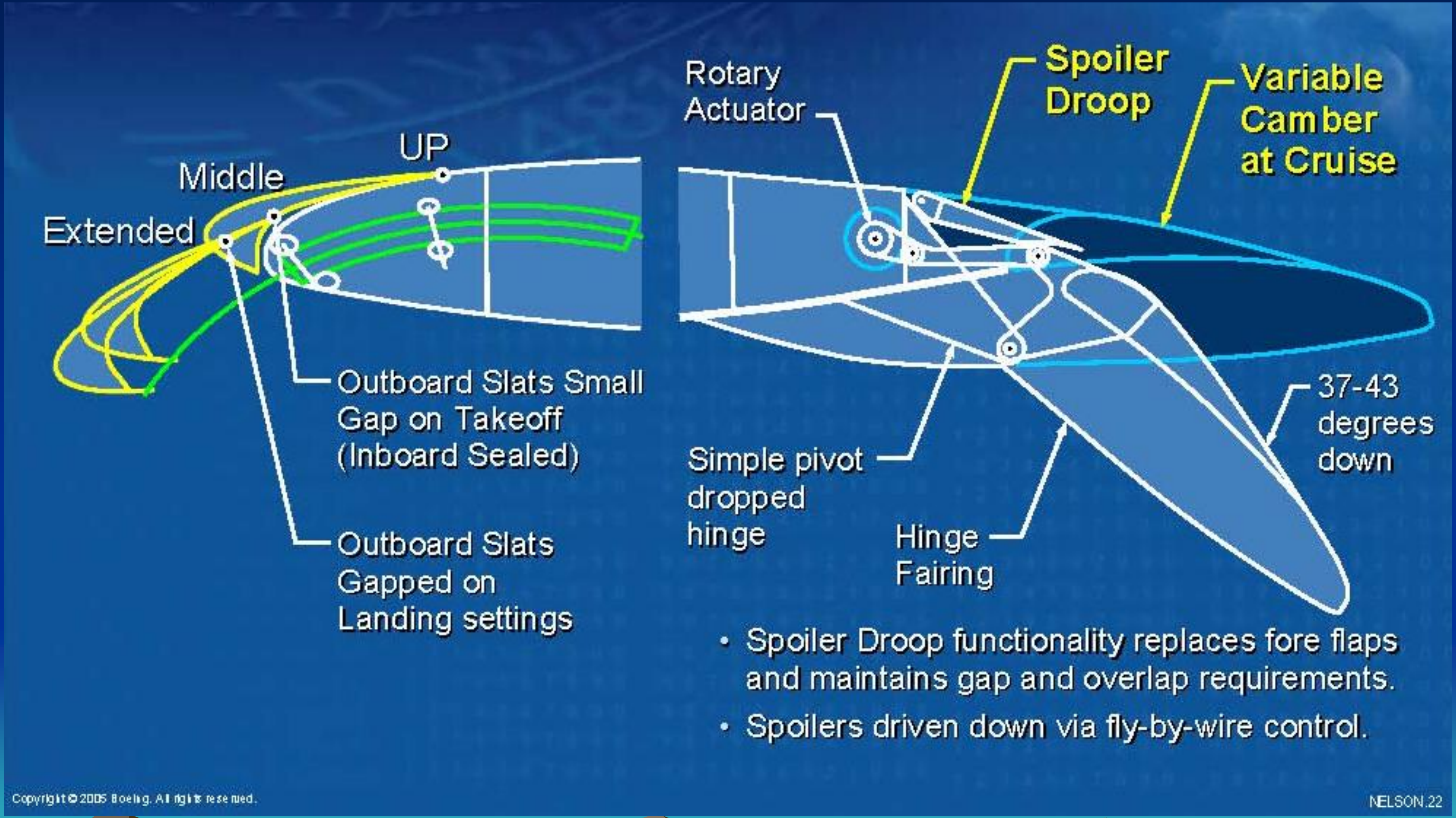
- Uses simple hinged flap with limited Fowler action
- Similar principle used on DC-10 and B787

747 Variable Camber Krüger Flap System



- Complex mechanical linkage

B787 Flap System



High Lift Devices

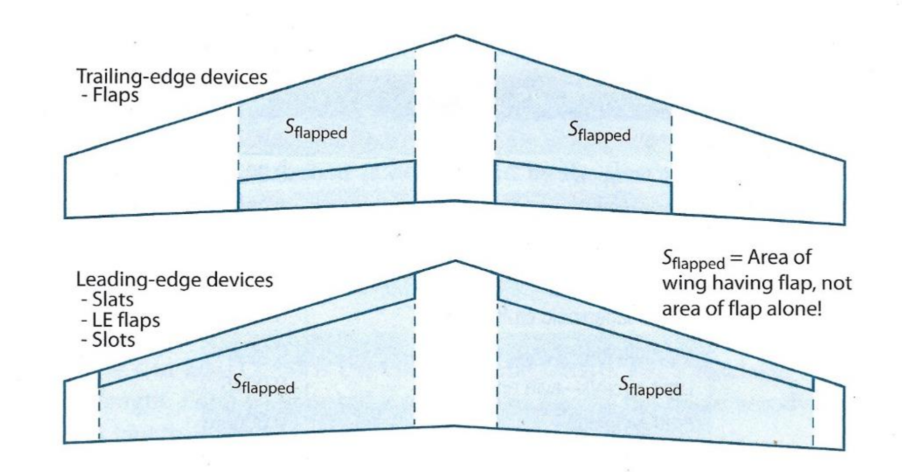
Raymer Eq. 12.21

$$\Delta C_{L_{max}} = 0.9\,\Delta C_{l_{max}} \left(\frac{S_{flapped}}{S_{ref}} \right) \cos \Lambda_{H.L.}$$

Raymer Eq. 12.22

$$\Delta \alpha_{OL} = \left(\Delta \alpha_{OL} \right)_{airfoil} \left(\frac{S_{flapped}}{S_{ref}} \right) \cos \Lambda_{H.L.}$$

$_{H.L.}$ = hinge line



High Lift Device	$\Delta C_{l_{max}}$
Flaps	
Plain and split	0.9
Slotted	1.3
Fowler	1.3 c'/c
Double slotted	1.6 c'/c
Triple slotted	1.6 c'/c
L.E. Devices	
Fixed slot	0.2
L.E. flap	0.3
Krűger flap	0.3
Slat	0.4 c'/c

Raymer Table 12.2

Lift and High Lift Systems

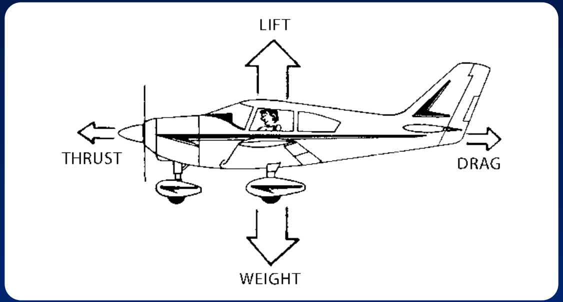
Zero-Lift Drag C_{D0}

Drag due to Lift C_{Di}

Wave Drag due to Volume $C_{D0_{supersonic}}$

Wave Drag due to Lift C_{DW}

Drag Polar



$$C_D = C_{D_0} + C_{D_i}$$

$$C_D = C_{D_0} + \frac{1}{\pi AR\, e} C_L^2$$

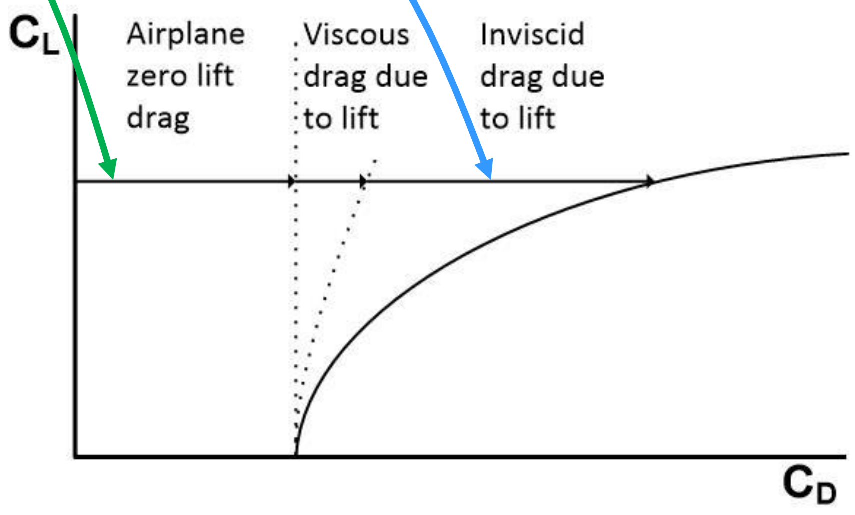
$$C_D = C_{D_0} + K C_L^2$$

where

$e =$ Oswald efficiency factor

$K =$ Drag-due-to-lift factor

C_{D_i} includes viscous drag due to lift



Need these two values

Drag Terminology Matrix

Zero-lift drag, C_{D_O}

Drag due to lift, C_{D_i} (usually, but erroneously, called “induced drag”)

	Shear forces	Pressure forces		
		Separation	Shock	Circulation
Parasite drag	Skin friction Scrubbing drag	Viscous separation	Wave drag	
		Shock-induced separation “drag rise”		
	Interference drag			
	Profile drag			
		Camber drag		
Drag due to lift [f(lift)]	Supervelocity effect on skin friction	Supervelocity effect on profile drag—e.g., landing gear		Induced drag Trim drag Wave drag due to lift
Reference area	S_{wetted}	Max. cross-section	(Volume distribution)	S_{ref}

Wave drag

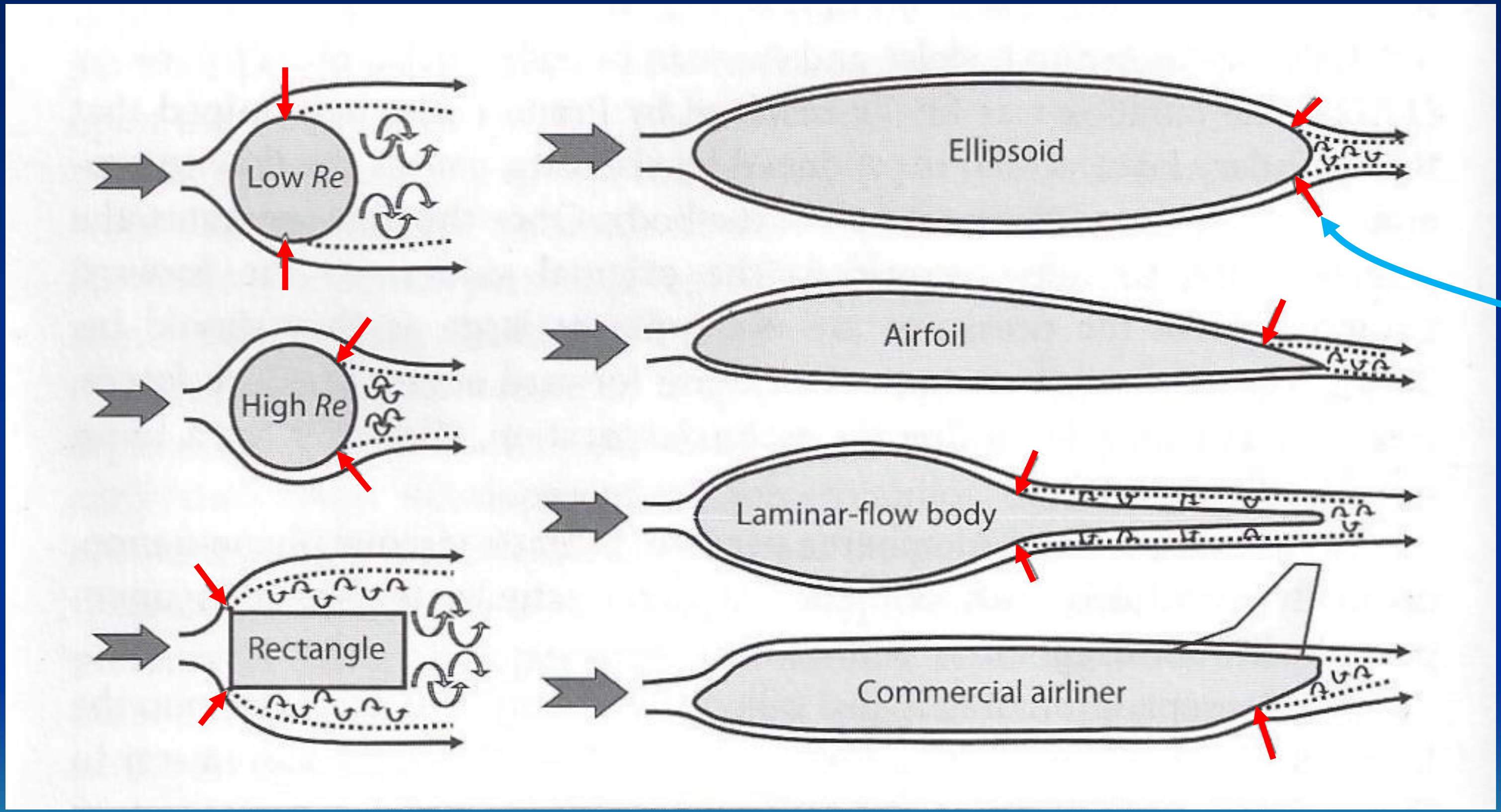
Includes boundary layer displacement

Inviscid drag

Wave drag

Source: Raymer

Drag of Bodies



Pressure on surface is that of fluid outside boundary layer

Arrows indicate separation location

Even if flow does not separate, pressure forces do not sum to zero

Potential flow analysis could predict lift, but not drag (d'Alembert's paradox, 1752)

- Two methods for calculating subsonic zero-lift drag
  - Equivalent skin friction method (approximate)
  - Component drag build-up method
    1. Streamlined components
       - Skin friction
       - Form
       - Interference
    2. Bluff components
    3. Leakage and protuberances

What is a “Drag Count”?

- Usually used in terms of zero-lift drag
- One drag count = ΔC_{D_o} x 10^4
  - i.e. one drag count is equivalent to $\Delta C_{D_o} = 0.0001$
- Why this value?
  - Because this is the smallest value of drag coefficient that can measured with confidence
- For a jet transport $C_{D_o} \approx 250$ counts

Equivalent Skin Friction Method

Equivalent Skin Friction Method:

For a flat plate with surface parallel to flow

$$D = C_f \, q \, S$$

where

C_f = skin friction coefficient

S = area

Note changed reference area

For an airplane

$$D_o = C_{f_e} \, q \, \boxed{S_{wet}}$$

where

C_{f_e} = $\boxed{\text{equivalent}}$ skin friction coefficient

S_{wet} = airplane wetted area

$$C_{D_o} = \frac{D_o}{q S_{ref}} = C_{f_e} \frac{S_{wet}}{S_{ref}}$$

Aircraft type	C_{f_e}
Civil transport	0.0026
Bomber	0.0030
Military cargo	0.0035
Air Force fighter	0.0035
Navy fighter	0.0040
Supersonic cruise aircraft	0.0025
Light aircraft - single engine	0.0055
Light aircraft - twin engine	0.0045
Seaplane - propeller driven	0.0065
Seaplane - jet	0.0040

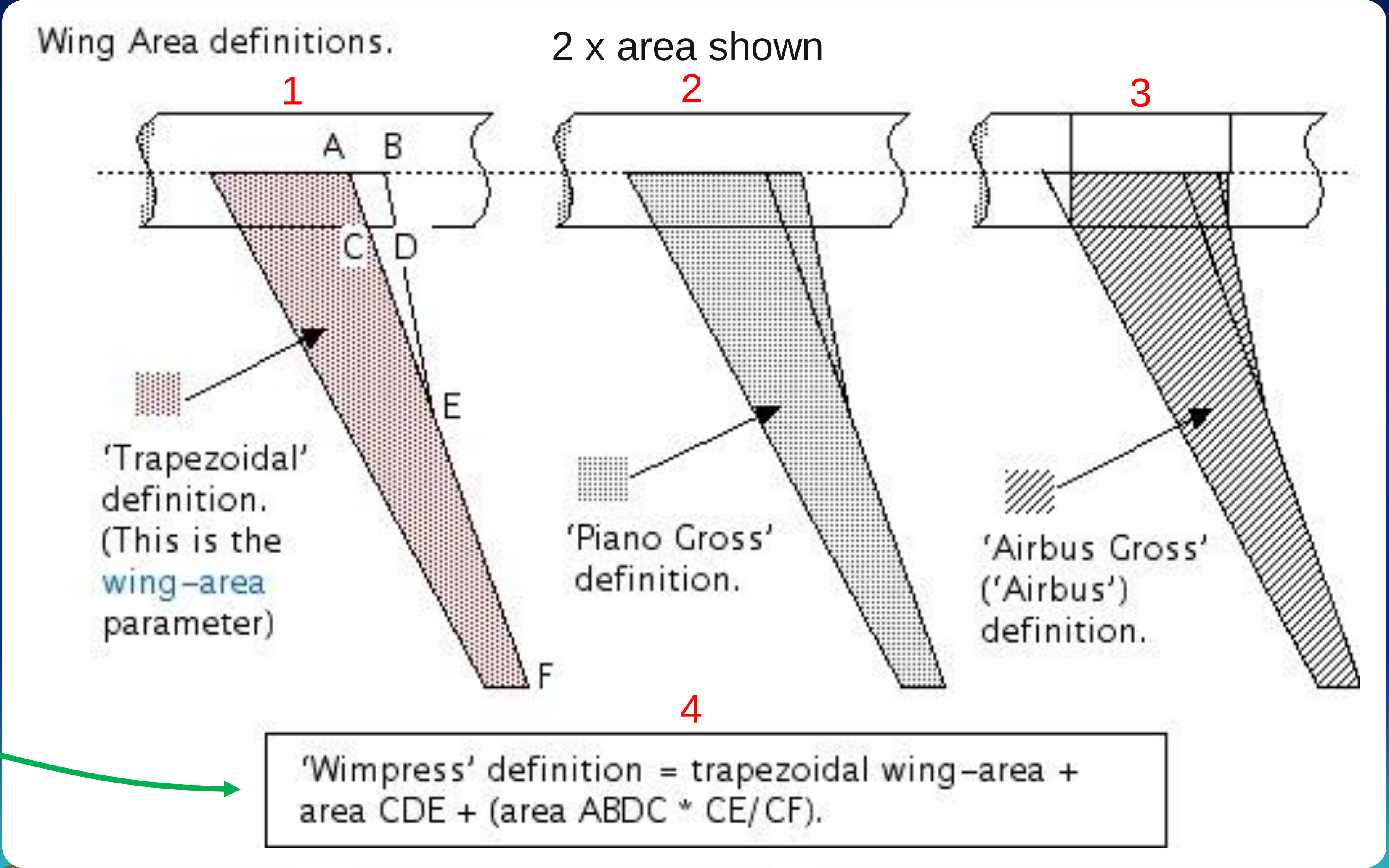
Source: Raymer (with modification)

Wing Reference Area Definitions

Four definitions here

For L.1011,
S_{ref} = 3456 ft^2
a number selected by the head of aerodynamics

Named after John Wimpress, chief aerodynamicist of B.767



Source: http://www.lissys.demon.co.uk/

Component Drag Build-up Method

- Also called “parasite” drag (because you can’t get rid of it)
- Defined as

$$C_{D_o} = C_{D_{streamlined}} + C_{D_{misc}} + C_{D_{L\&P}}$$

where

$C_{D_{streamlined}}$ = Zero lift drag coeff due to streamlined components

$C_{D_{misc}}$ = Zero lift drag coeff due to misc bluff assemblies

$C_{D_{L\&P}}$ = Zero lift drag coeff due to leakage and protuberances

Component Definitions

- Streamlined components are defined as objects for which skin friction drag dominates (e.g., wing, fuselage, horizontal and vertical tail, nacelles, pylons, etc.)
- Miscellaneous components are defined as bluff objects for which pressure drag dominates (e.g., wheels and struts, wire bracing, hemispherical protrusion on side, top, or bottom of fuselage, etc.)

Flat Plate Skin Friction Coefficient

For laminar flow

$$C_f = \frac{1.328}{\sqrt{R_n}}$$

For turbulent flow

$$C_f = \frac{0.455}{\left(\log_{10} R_n\right)^{2.58}\left(1 + 0.144M^2\right)^{0.65}}$$

where

$$R_n = \frac{\rho V l}{\mu}$$

l = characteristic length, i.e.

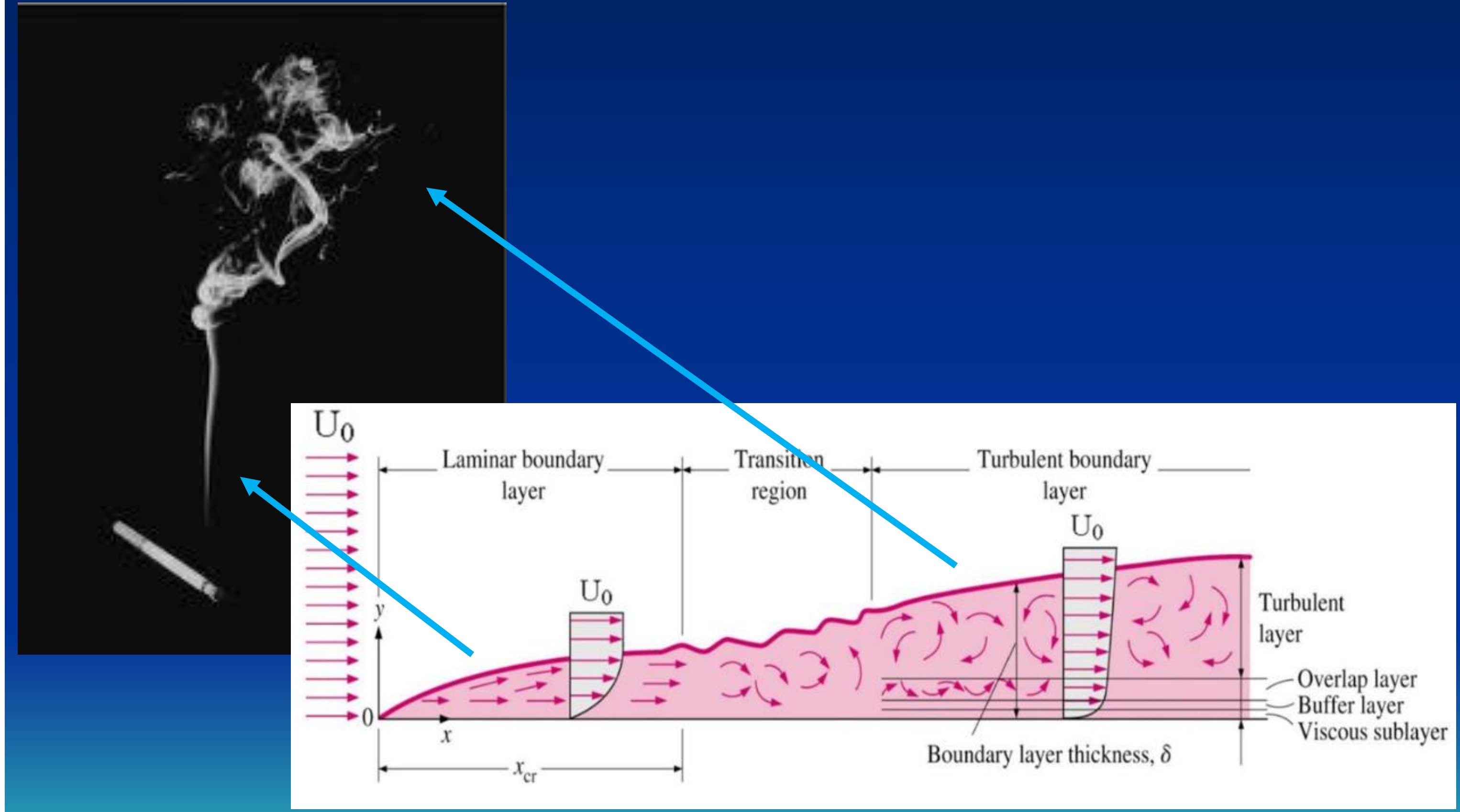
- mac of lifting surface,
- length of fuselage
- average chord of pylon

ρ = fluid density

V = freestream velocity

μ = kinematic viscosity

For large airplanes, flow is nearly always turbulent



Drag of Streamwise Flat Plate

Skin friction drag

$$D = C_f \left(\frac{1}{2}\right) \rho V^2 S_{wet} = C_f \, q \, S_{wet}$$

where

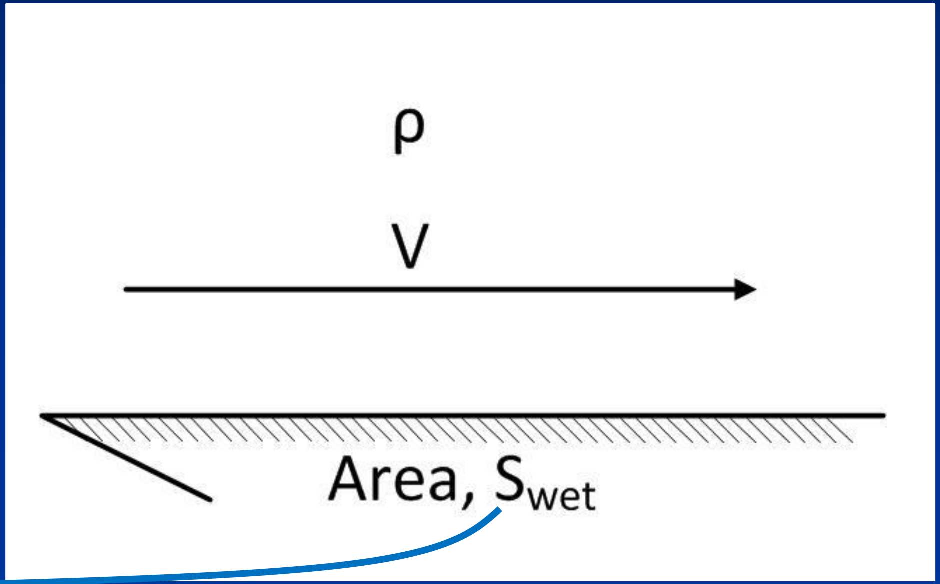
C_f = skin friction coefficient

Divide by q

$$\frac{D}{q} = C_f \, S_{wet}$$

For complete aircraft

$$\left(\Delta C_{D_0}\right)_{\text{flat plate}} = \frac{D}{q \, S_{ref}} = C_{f_{\text{flat plate}}} \frac{S_{wet}}{S_{ref}}$$



Airplane reference wing area
(if you put the same flat plate on a different airplane, the value of $(\Delta C_{D_0})_{\text{flat plate}}$ will be different)

Summing Values of C_{D_o}

Considering skin friction only,

the sum of $\left(C_{D_0}\right)_c$ for all components

would be $\sum_{c=1}^{n} \frac{C_{f_c} S_{wet_c}}{S_{ref}}$

where subscript c refers to an aircraft component

n = number of components

By including b.l. displacement effects,

we must deal with form drag and interference drag.

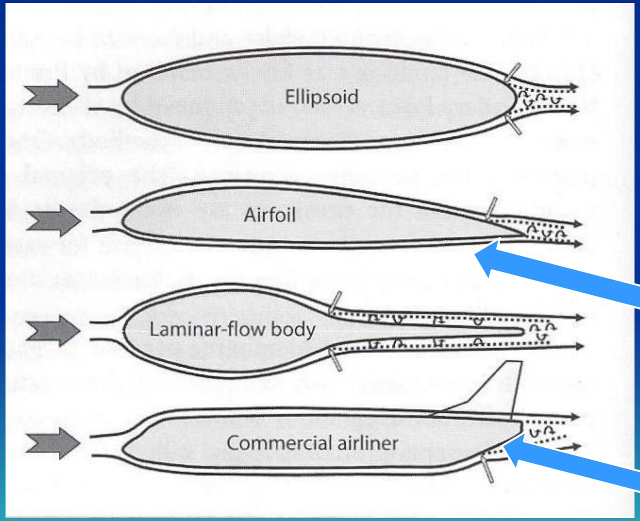
For each component, c, we factor the

value of $\left(C_{D_0}\right)_c$ by an empirical form factor, FF_c,

and (where appropriate) an empirical interference factor Q_c

So

$$\sum \left(C_{D_0}\right)_{comp} = \sum_{c=1}^{n} \frac{\left(C_{f_c} S_{wet_c} FF_c Q_c\right)}{S_{ref}}$$



Source: Raymer

Boundary layer growth: pressure distribution is that of a body that is not closed (i.e. resolving D'Alembert's Paradox).

Aggravated if separation occurs

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Form Factors

For wing, tail, strut and pylon

$$FF = \left(1 + \frac{0.6}{\left(\frac{x}{c}\right)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right)\left(1.34\,M^{0.18}\left(\cos\Lambda_m\right)^{0.28}\right)$$

where

$\left(\frac{x}{c}\right)_m$ = chordwise location of the airfoil maximum thickness point

$\left(\frac{t}{c}\right) = \text{average } \frac{\text{thickness}}{\text{chord}} \text{ ratio}$

Λ_m = sweep of the maximum thickness line

For fuselage and smooth canopy

$$FF = \left(1 + \frac{60}{f^3} + \frac{f}{400}\right)$$

For nacelle and smooth external store

$$FF = 1 + \frac{0.35}{f}$$

where

f = fineness ratio, defined as

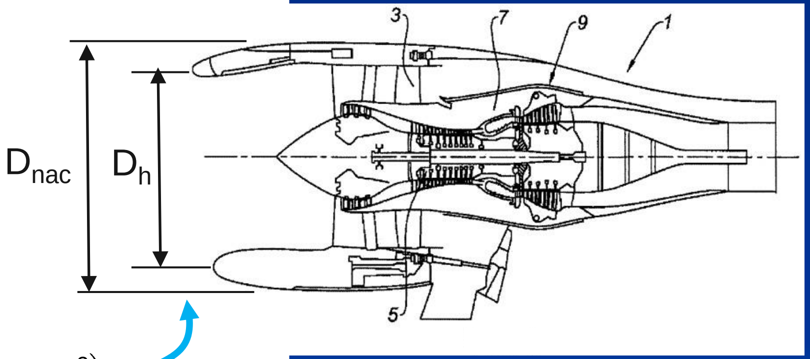
$$f = \frac{l}{d} = \frac{l}{\sqrt{\frac{4}{\pi} A_{max}}}$$

where

l = component length

d = component diameter

For a nacelle $A_{max} = \frac{\pi}{4}\left(D_{nac}^2 - D_h^2\right)$



D_{nac} = nacelle max diameter

D_h = nacelle highlight diameter

Interference Factors

Condition	Q
Nacelle or external store mounted directly on fuselage or wing	1.5
Nacelle or external store less than one diameter from fuselage or wing	1.3
Nacelle or external store more than one diameter from fuselage or wing	1.0
Wingtip-mounted missiles	1.25
High wing, mid wing or well-filleted low wing	1.0
Unfilleted low wing	1.1-1.4
Conventional tail	1.04-1.05
V-tail	1.03
H-tail	1.08

Source: Raymer

For more information see Hoerner Chapter VIII Interference Drag

Aero Drag of Floats and Hulls

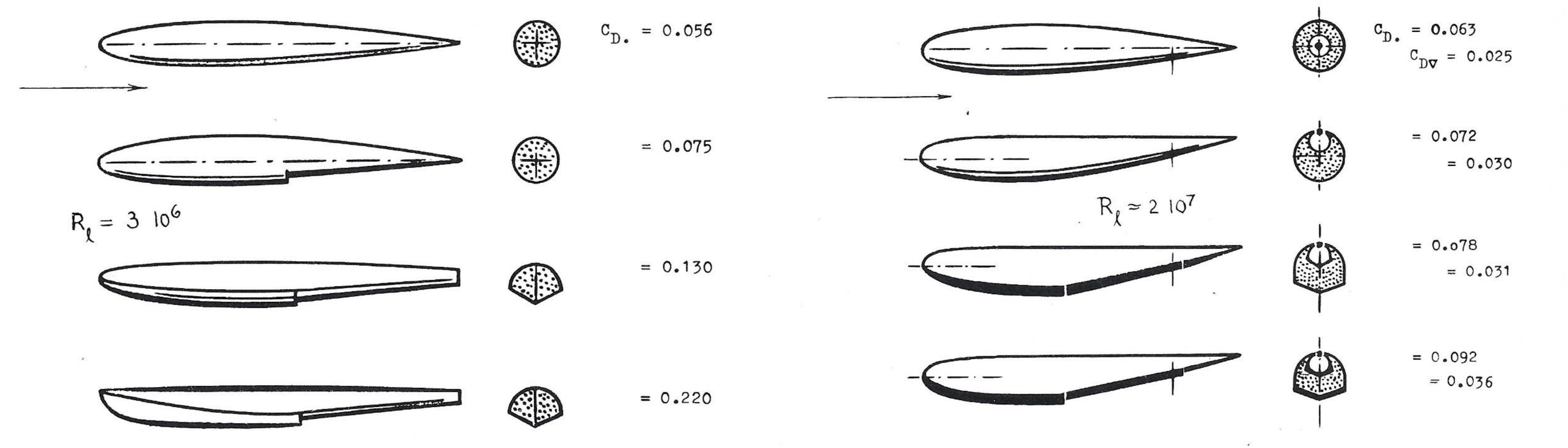


Figure 22. Drag of a float (12,a) developed from a basic streamline body by adding step and chines.

Figure 23. Drag of flying-boat hull (14,a), developed from streamline body having same length and same displacement.

Source: Hoerner

Drag coefficient based on maximum cross-section area

For more information see Hoerner Chapter XIII Drag of Aircraft Components

Hydro Drag of Floats

Drag decreases dramatically once floats start to plane and some wing-borne lift is achieved

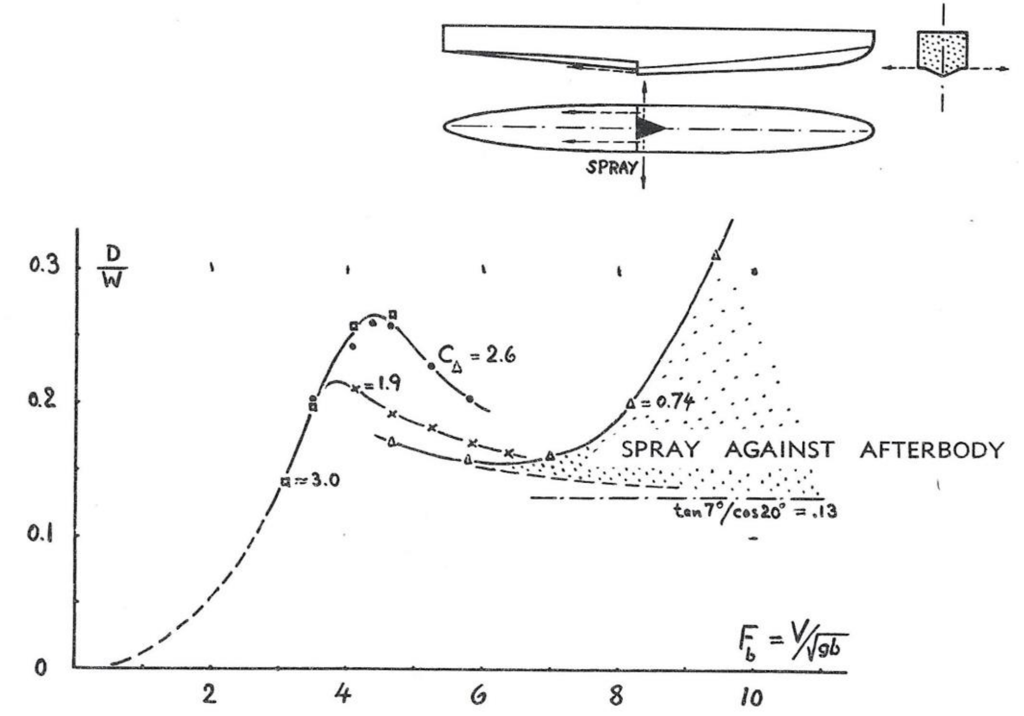


Figure 28. Drag-weight ratio of an airplane float (32,a) as a function of Froude number. Float data: DVL No.7, at $\alpha = 7°$ = constant, $l/b = 9.2$, $b = 0.3$ m. Coefficient $C_\Delta = W/\gamma b^3$.

Source: Hoerner

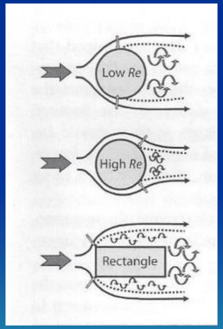
For more information see Hoerner Chapter XI Resistance of Water-Borne Craft

Miscellaneous Components

Calculate component $\frac{D}{q}$ based on frontal area

Sum the values of $\frac{D}{q}$ and divide by airplane reference area

$$C_{D_{msic}} = \sum_{c=1}^{n} \left(\frac{D}{q}\right)_c \frac{1}{S_{ref}}$$



Source: Raymer

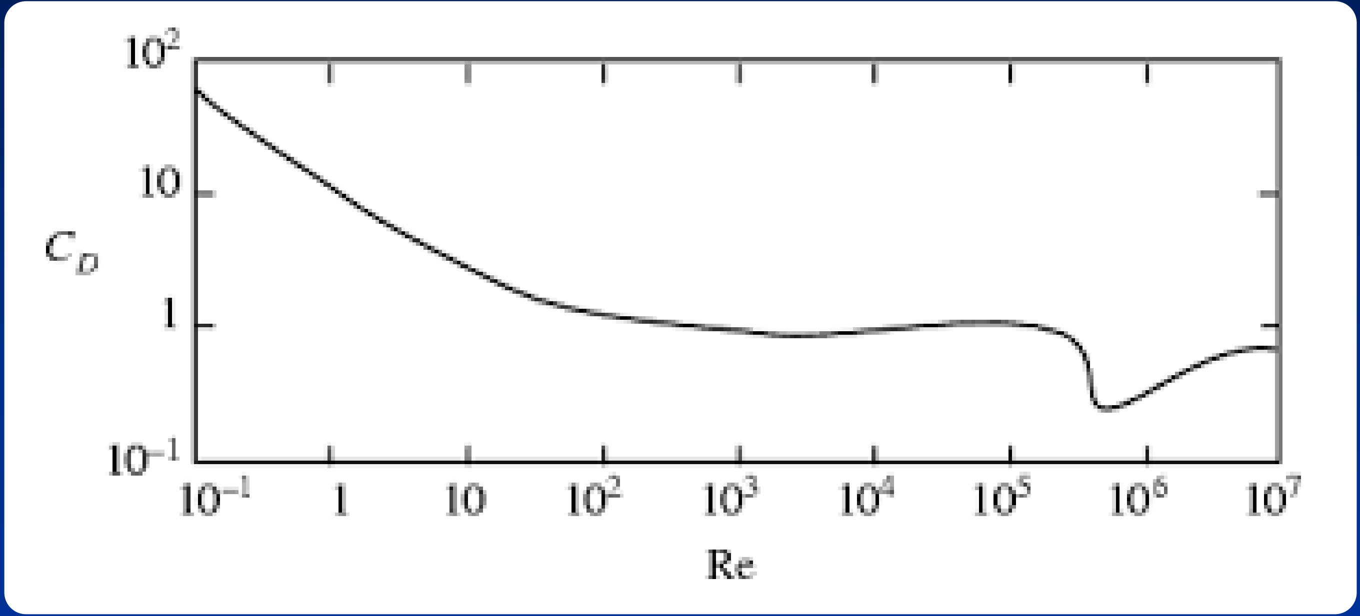
Component	D/q per unit frontal area
Wheel and tire	0.25
Second wheel in tandem	0.15
Streamlined wheel and tire	0.18
Wheel and tire with fairing	0.13
Streamlined strut (0.17<t/c<0.33)	0.05
Round strut or wire	0.30 *
Flat spring gear leg	1.40
Fork, bogey, irregular fitting	1.0-1.4

* If subcritical, use D/q = 1.2

Multiply these values by <u>frontal</u> area to obtain D/q for that component

For more information see Hoerner Chapter XIII Aircraft Components

Cylinder Drag is R_e - dependent



Detailed Flap Drag

Two components

— due to separated flow

— due to change in span loading

Flap drag due to separated flow

$$\Delta C_{D_{flaps}} = F_{flap}\left(\frac{c_{flap}}{c}\right)\left(\frac{S_{flapped}}{S_{ref}}\right)\left(\delta_{flap} - 10\right)$$

where

δ_{flap} = flap deflection in degrees

F_{flap} = 0.0144 for plain flaps

F_{flap} = 0.0074 for slotted flaps

c_{flap} = chord length of flap

Raymer Eq.(12.61)

Boeing 727 flaps

$$\Delta C_{D_i} = k_f^2\left(\Delta C_{L_{flap}}\right)^2 \cos\Lambda_{\frac{c}{4}}$$

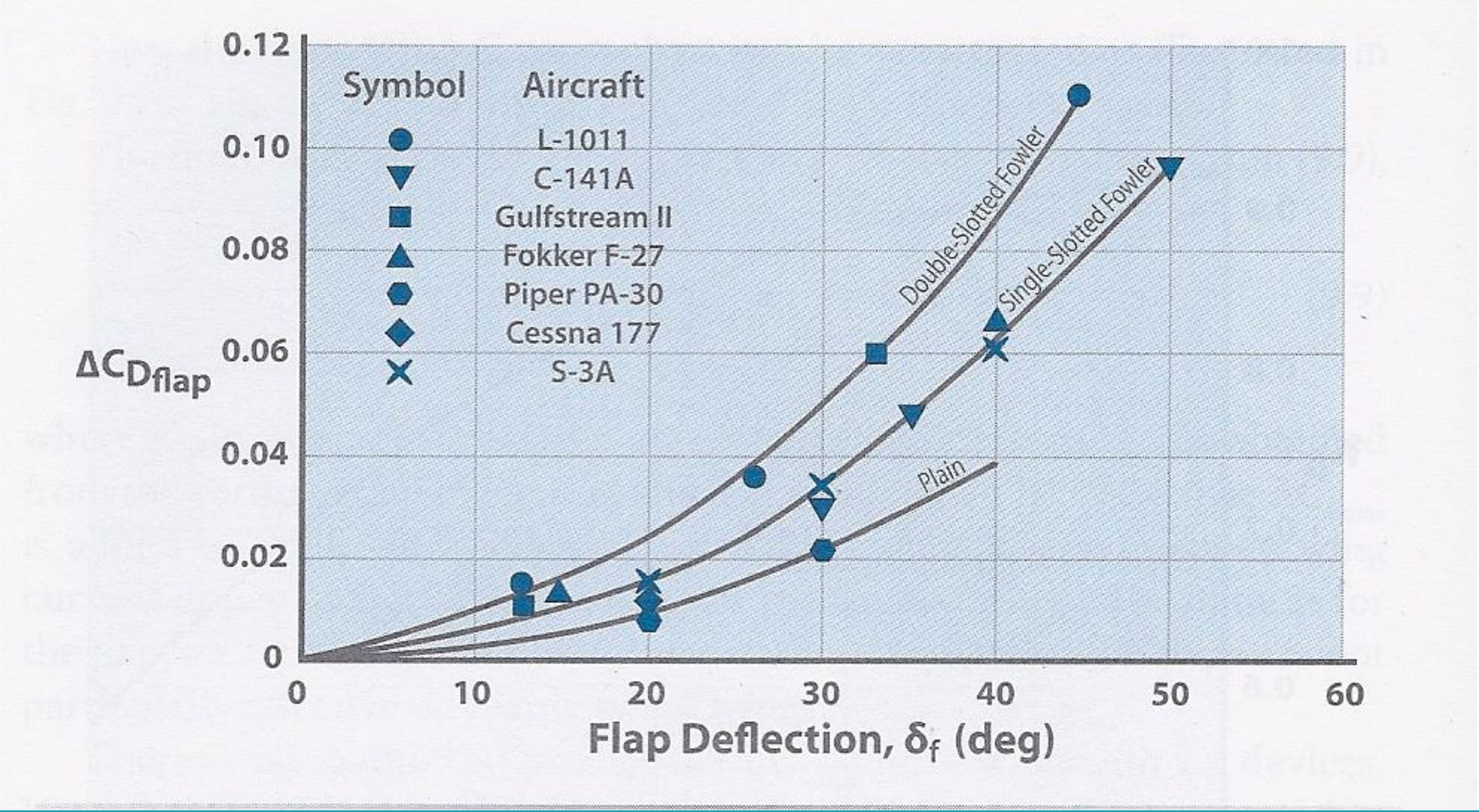
k_f = 0.14 for full span flaps

= 0.28 for half span flaps

Raymer Eq.(12.62)

Approximate Flap Drag

$\Delta C_{D_{flap}}$ referenced to wing area



Source: Nicolai/Carichner

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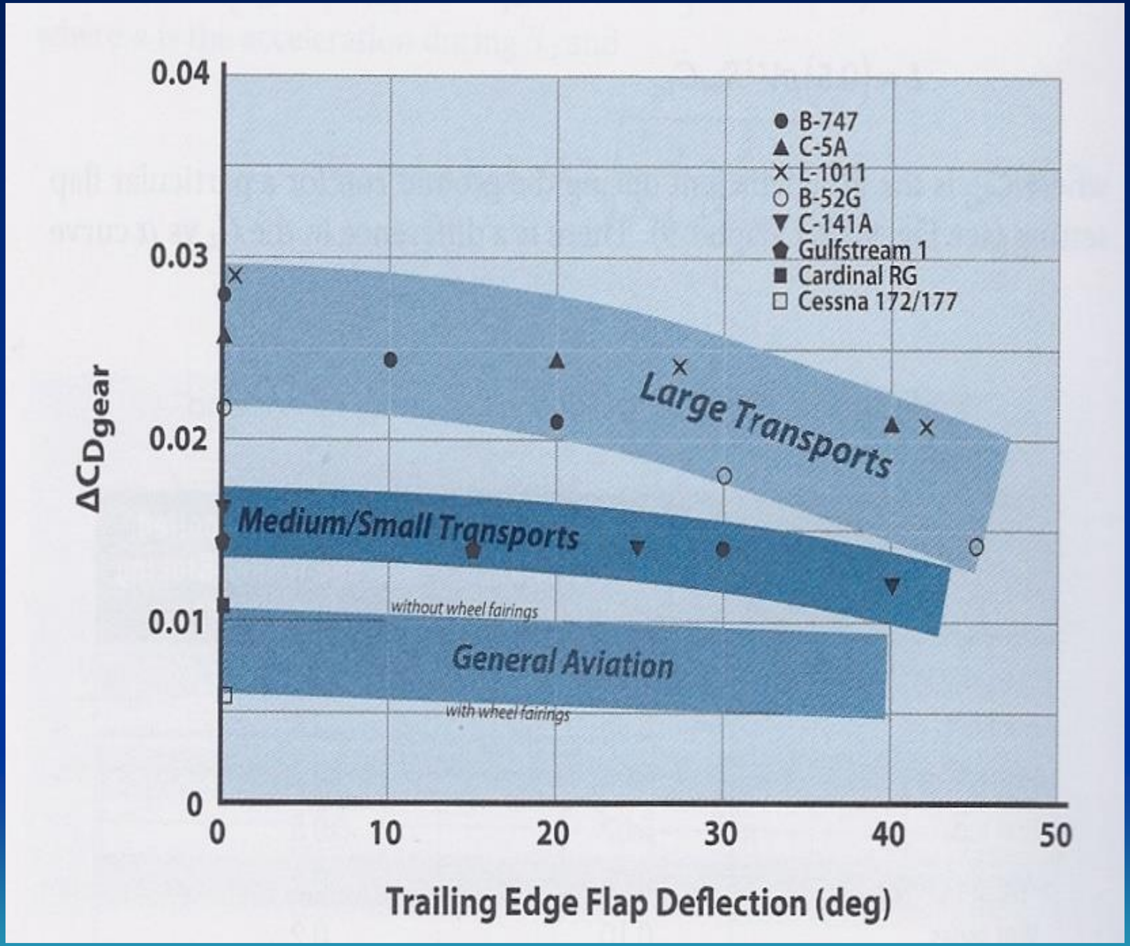
Approximate Landing Gear Drag

Usually calculate landing gear drag by component, and verify with wind tunnel tests

Use this figure for ball-park check ($\Delta C_{D_{gear}}$ referenced to wing area)

Why does drag decrease when flaps are deflected?



Source: Nicolai /Carichner

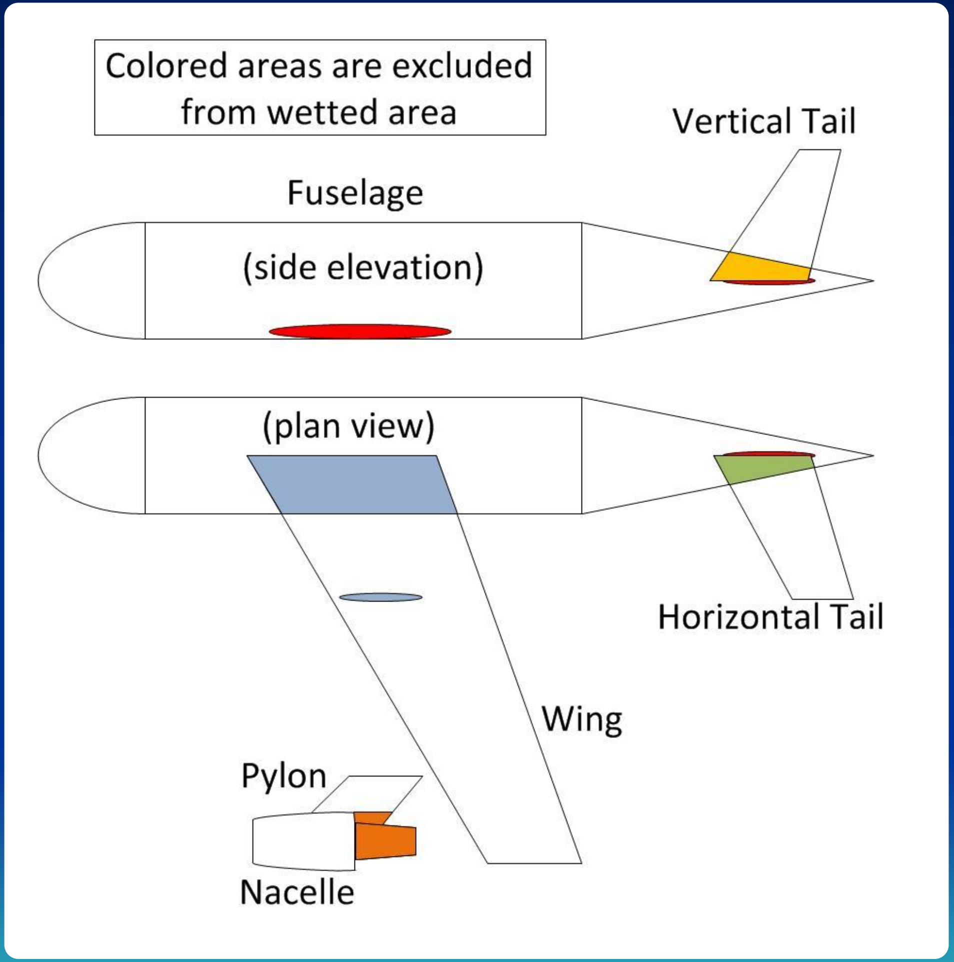
Leakage and Protuberance Drag

- Caused by
  - air entering airframe in high surface pressure areas (increased momentum drag)
  - air exiting airframe in low surface pressure areas (increased separation drag)

Category	$C_{D_{L\&P}}$
Bombers or jet transports	2-5%
Propeller-driven	5-10%
Current fighters	10-15%
Next-gen fighters	5-10%

Scaling Lifting Surfaces and Nacelles

- In mission sizing program some parts must be rescaled on every weight iteration
  - wing
  - horizontal tail
  - vertical tail
  - nacelles



Spreadsheet Geometry Module

- Given T/W and W/S
- Assume W_0
- So T and S known
- From assumptions on non-dim. geometry can calculate dimensional data

Wing	Horiz Tail	Vert Tail	Pylon	Fuselage	Nacelles
AR_{wing}	AR_{ht}	AR_{vt}	l_{pylon}/d_{nac}	l_{fuse}	$l_{ref\text{-}nac}$
Λ_{wing}	Λ_{ht}	Λ_{vt}	c_{pylon}/d_{nac}	d_{fuse}	$d_{ref\text{-}nac}$
λ_{wing}	λ_{ht}			l_{taper}	
t/c_{wing}	t/c_{ht}	t/c_{vt}			
	$\bar{V}_{ht}$	$\bar{V}_{vt}$			
S_{wing}	S_{ht}	S_{vt}	l_{pylon}	$S_{wet\text{-}gross}$	l_{nac}
mac_{wing}	mac_{ht}	mac_{vt}	c_{pylon}	$S_{wet\text{-}net}$	d_{nac}
$c_{wing\text{-}sob}$	$c_{ht\text{-}sob}$	$c_{vt\text{-}sob}$	$S_{pylon\text{-}wet}$		
$t_{wing\text{-}sob}$	$t_{ht\text{-}sob}$	$t_{vt\text{-}sob}$			
$A_{wing\text{-}sob}$	$A_{ht\text{-}sob}$	$A_{vt\text{-}sob}$			
$S_{wing\text{-}wet}$	$S_{ht\text{-}wet}$	$S_{vt\text{-}wet}$			

Non-dimensional geometry (except fuselage)

Dimensions for input to drag buildup

Zero-Lift Drag Module

Component	S_{wet}	S_{xs}	l_{ref}	R	C_f	FF	Q	$\frac{D/q}{S_{xs}}$	D/q	ΔC_{D_0}
Wing										
Horiz. Tail										
Vert Tail										
Pylons										
Fuselage										
Nacelles										
Landing gear										
Flaps+slats										
Total										$\Sigma\Delta C_{D_0}$

S_{wet} = wetted area S_{xs} = cross-section area l_{ref} = reference length R = Reynolds number

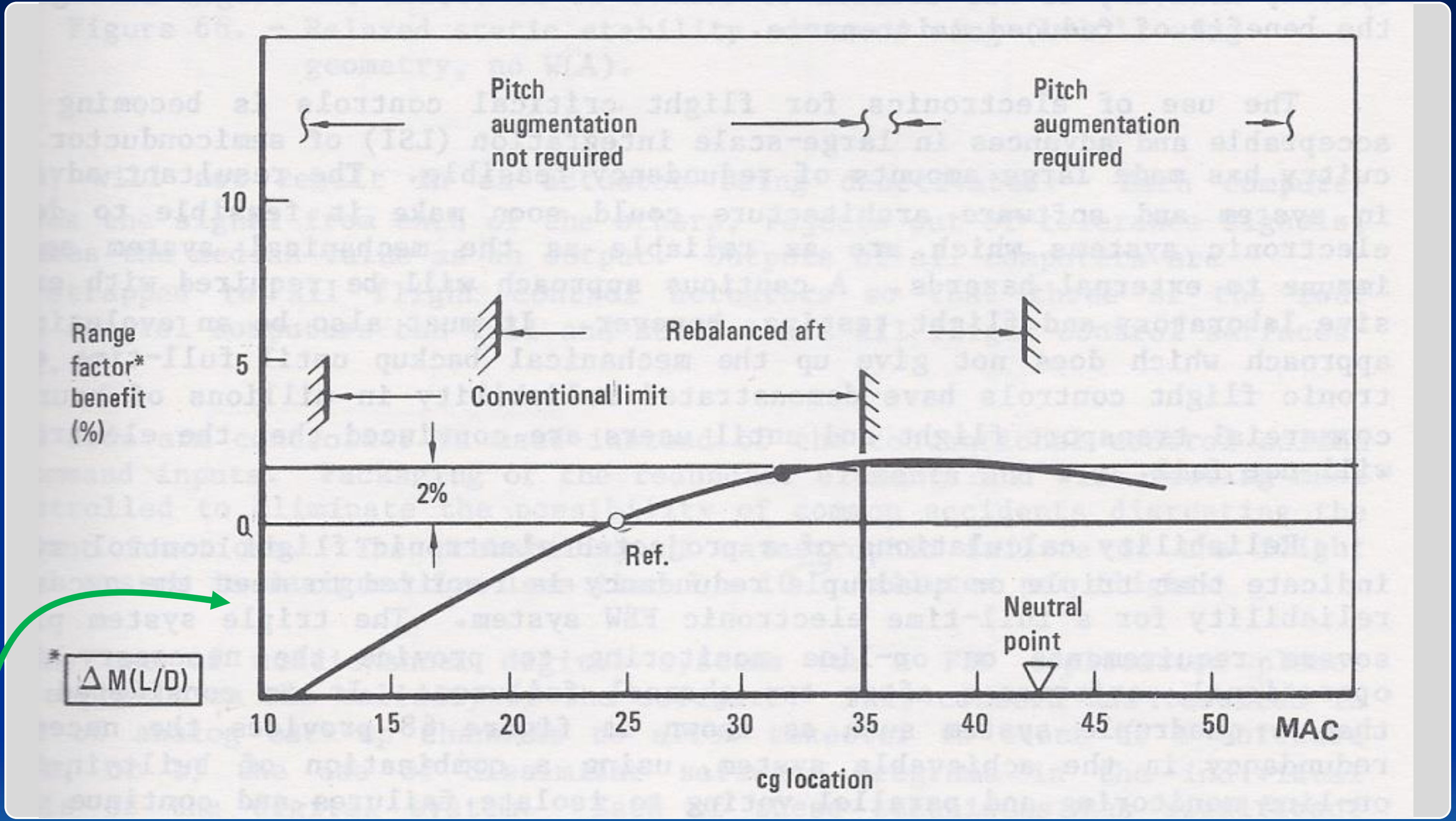
C_f = skin friction coeff Q = interference factor FF = form factor D/q = equivalent flat plate area

$\Delta C_{D_0} = (S_{wet}\, C_f\, Q\, FF)/S_{ref}$ or $\Delta C_{D_0} =$ D/q S_{ref}

Trim Drag

- Often approximated in conceptual design*
- Strong function of c.g. location
- Consists of
  - Induced drag of horizontal stabilizer
  - Drag of deflected elevator
  - Additional C_{D_i} due to additional wing lift

*Nicolai & Carichner (sec. 23.3.2) suggests trim drag is approx. 5% of total drag

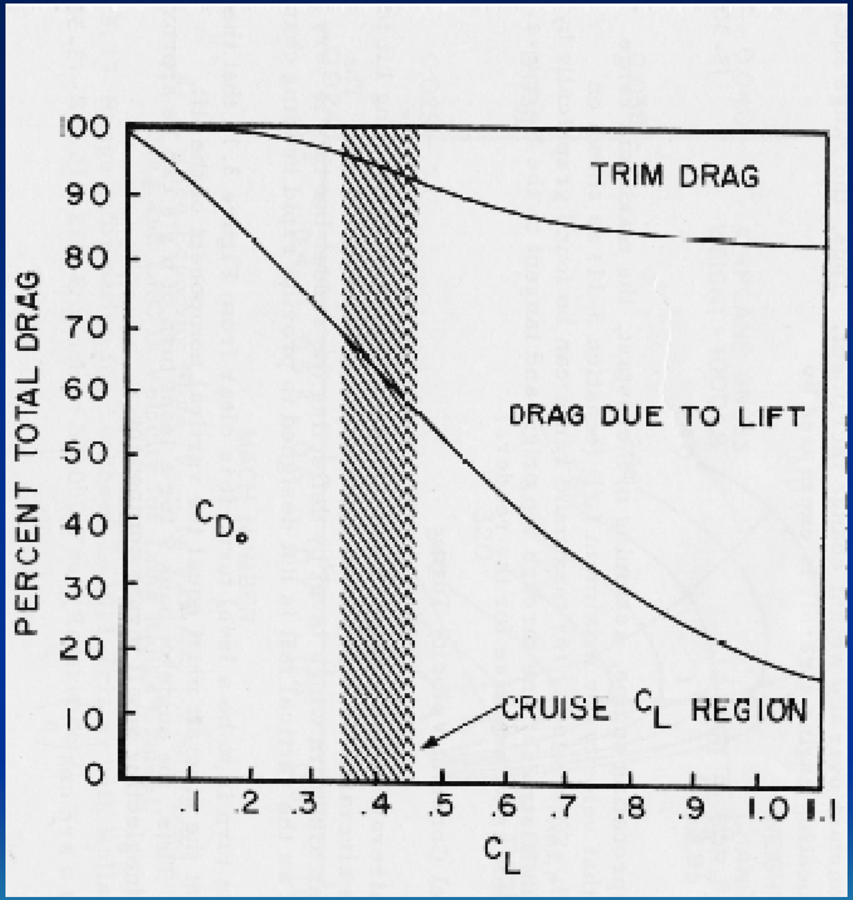


Effect of Relaxed Static Stability on L1011 Range Factor (NASA CR-3586)

Potential 6% difference in ML/D due to c.g. travel

Trim Drag

- If time is available, follow process in Raymer Sec. 16.3.10
- This assumes static margin (and thus c.g.) is fixed, which in practice is not the case
- Otherwise use Nicolai & Carichner value of 5% of total drag



Bill Mason, VTI, Config Aero Drag class notes

C-141 Drag Breakdown

Lift and High Lift Systems

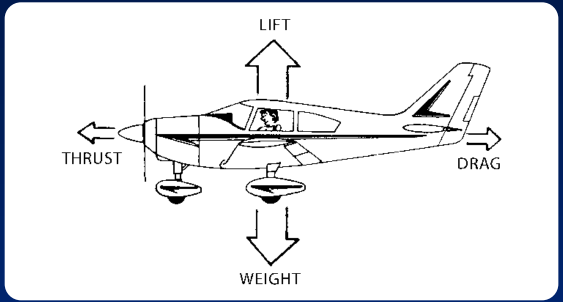
Zero-Lift Drag C_{D0}

Drag due to Lift C_{Di}

Wave Drag due to Volume $C_{D0_{supersonic}}$

Wave Drag due to Lift C_{DW}

Drag Polar



$$C_D = C_{D_0} + C_{D_i}$$

$$C_D = C_{D_0} + \frac{1}{\pi AR\, e} C_L^2$$

$$C_D = C_{D_0} + K\, C_L^2$$

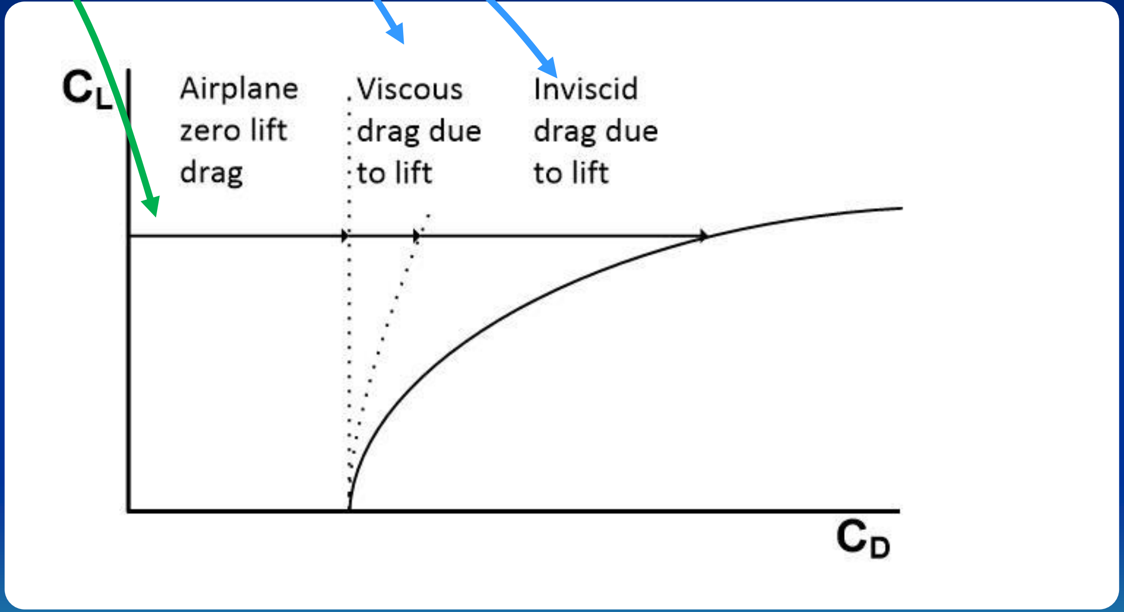
where

$e =$ Oswald efficiency factor

$K =$ Drag-due-to-lift factor

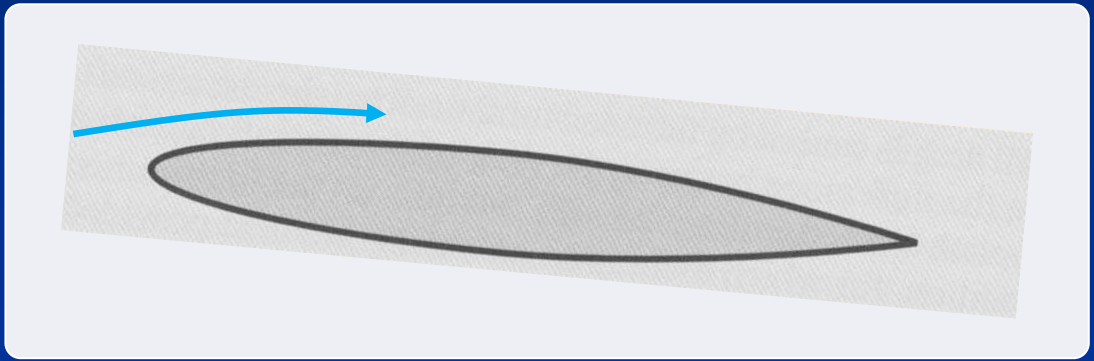
Need these two values

C_{D_i} includes viscous drag due to lift



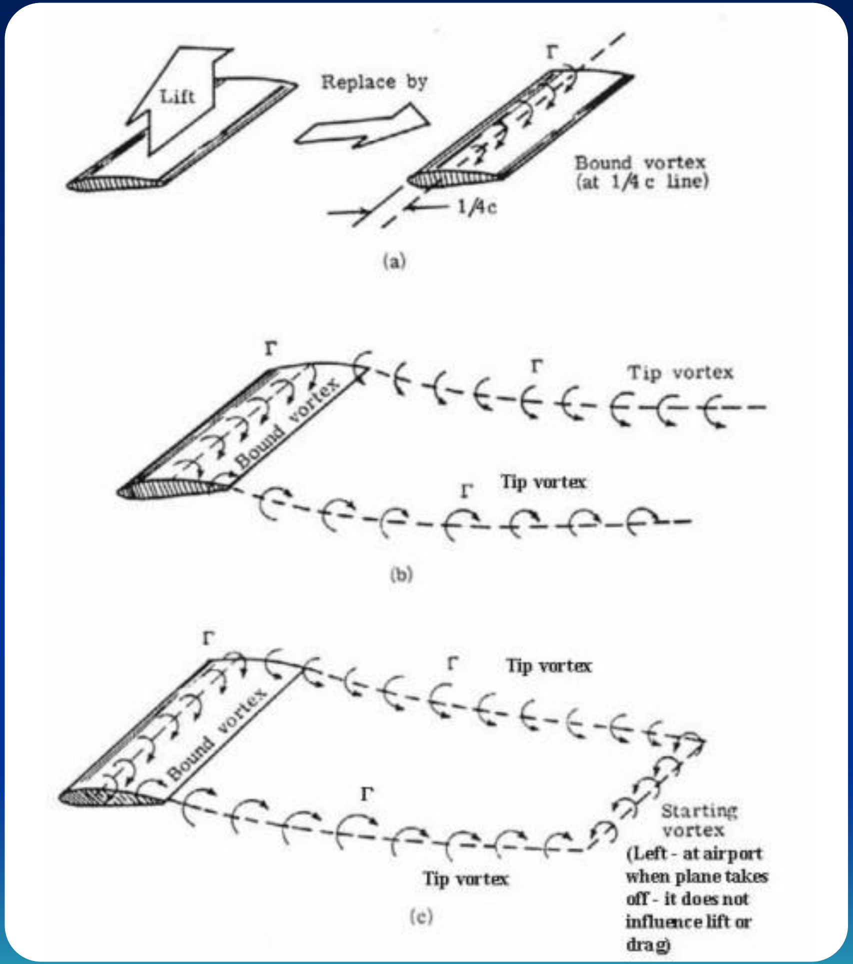
Viscous Drag due to Lift

Increased flow velocity on upper surface increase skin friction drag

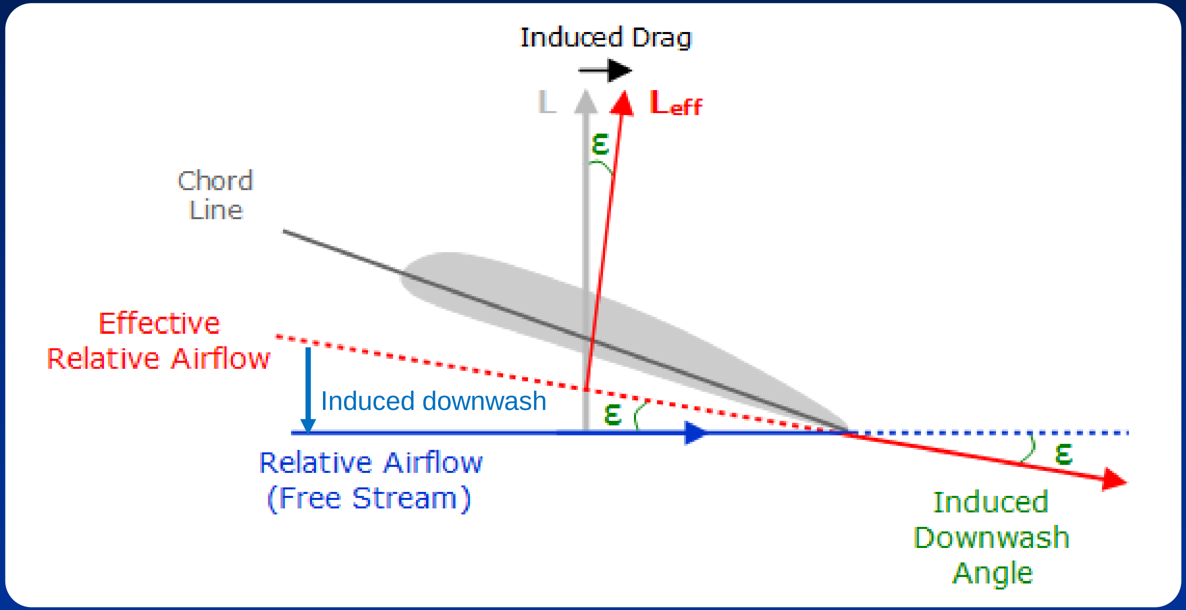


Drag-due-to-Lift Coefficient C_{D_i}

Inviscid flow theory



http://www.pilotfriend.com/training/flight_training/aero/wng_vort.htm



Source: Wikipedia

Drag due to lift factor

$$K = \frac{1}{\pi AR\, e}$$

Distribution of Circulation

Put spanwise location, y, in terms of θ where

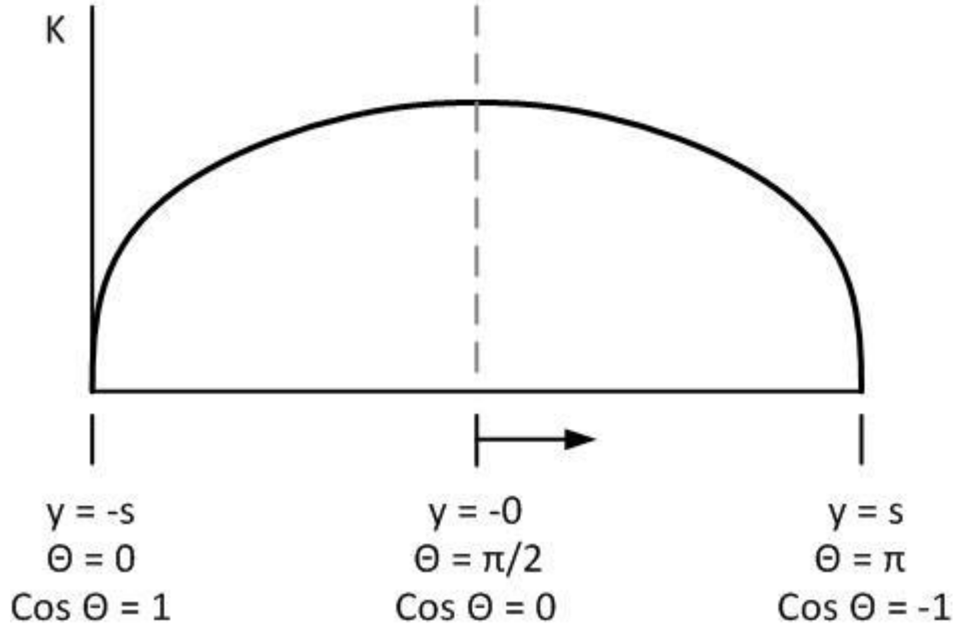
$$y = -s\cos\theta$$

Define spanwise distribution of circulation, Γ, as a Fourier series

$$\Gamma = -U\,4\,s\sum_{n=1}^{\infty} A_n \sin n\theta$$

Total lift

$$L = -\int_{-s}^{+s} \rho U \Gamma dy$$



Distribution of Circulation for Minimum D_i

All terms in Fourier series contribute to drag

so for minimum induced drag $A_2 = A_3 = A_4 = \ldots = 0$

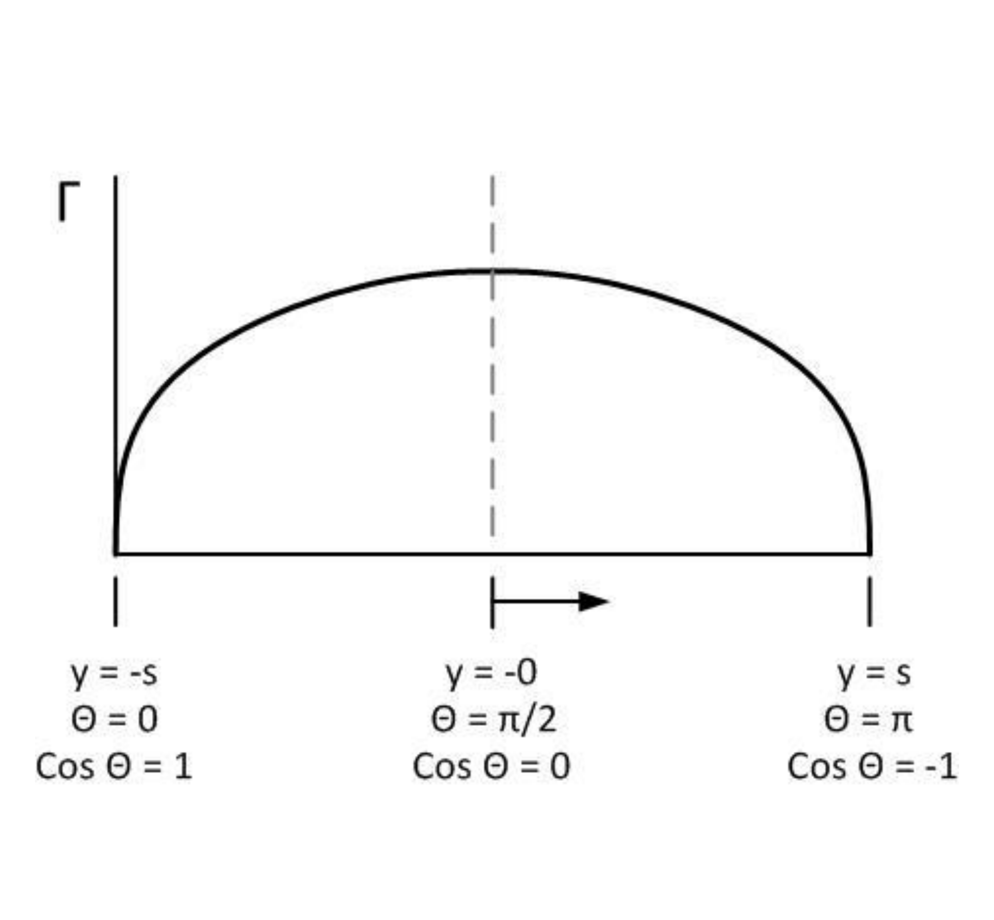
$$\Gamma = -4\,U\,s\,A_1 \sin\theta$$

$$\cos\theta = \frac{y}{s} \quad \text{so} \quad \sin\theta = \sqrt{1 - \frac{y^2}{s^2}}$$

$$\Gamma = -4\,U\,s\,A_1\sqrt{1 - \frac{y^2}{s^2}}$$

$$\left(\frac{\Gamma}{-4\,U\,s\,A_1}\right)^2 + \left(\frac{y}{s}\right)^2 = 1$$

i.e. spanwise elliptic distribution of Γ



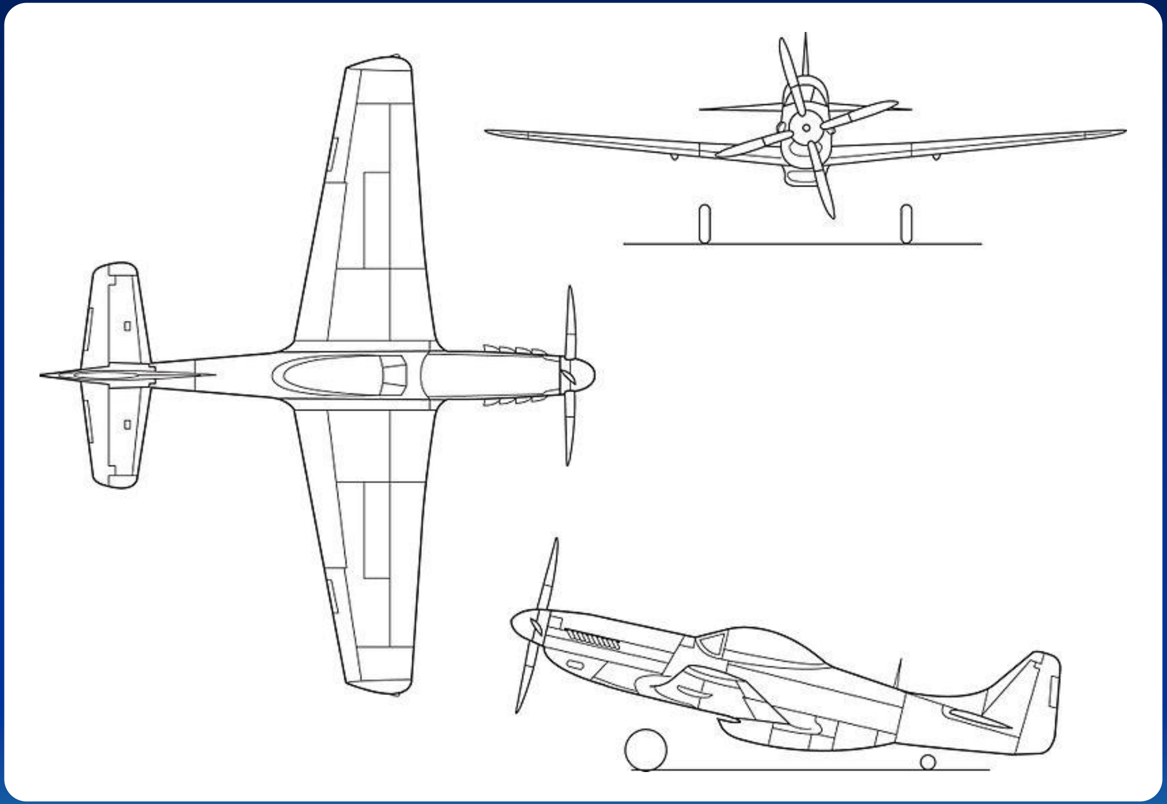
Planform with Minimum Induced Drag

Elliptical planform has minimum induced drag at all values of C_L

Spitfire vs. P.51 Comparison

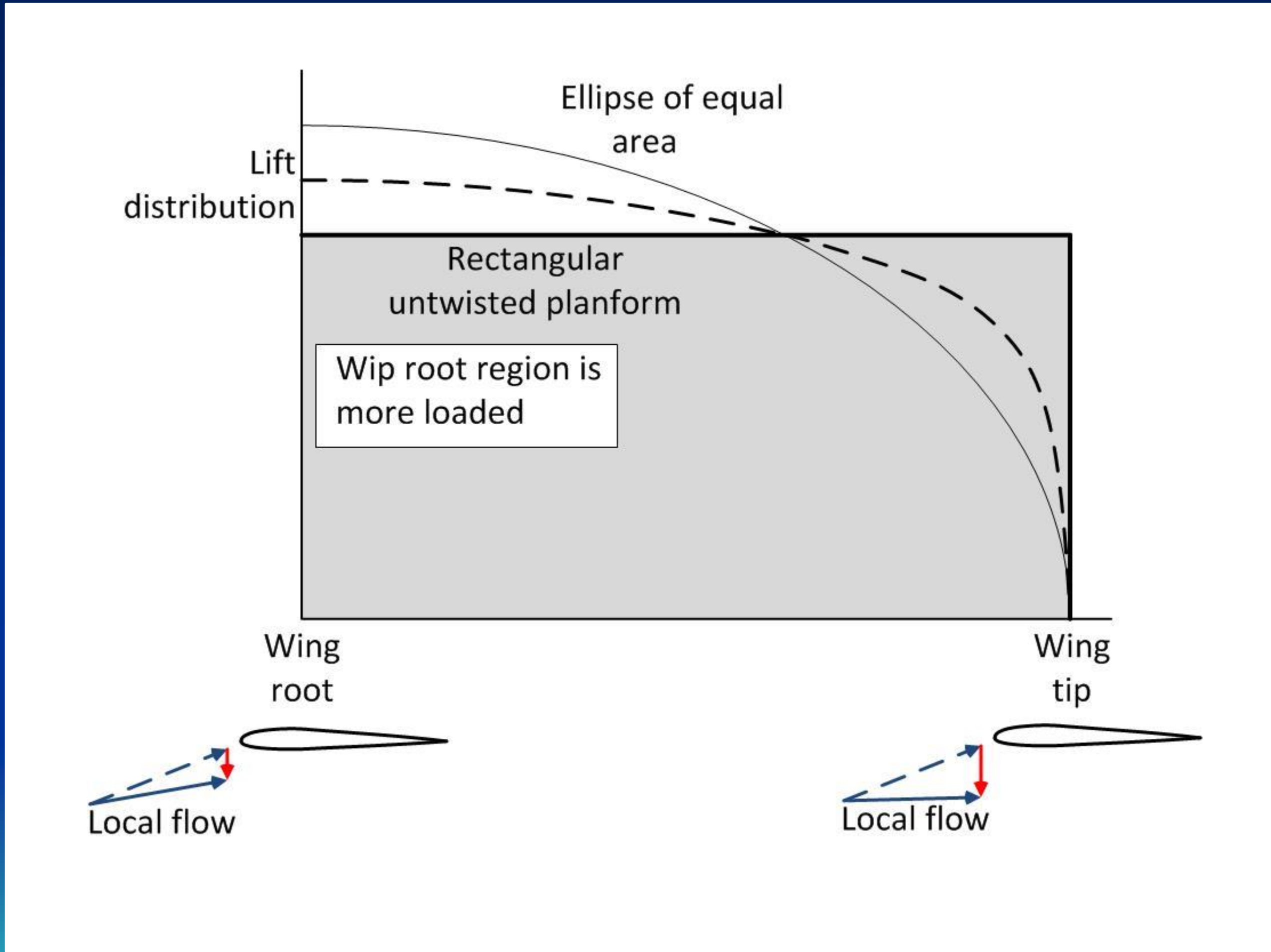
	Spitfire	P.51D
MTOGW – kg (lb)	6,700 (3,039)	12,100 (5,488)
EW – kg (lb)	5,065 (2,297)	7,635 (3,465)
EW/TOGW	0.76	0.63
Range – km (nmi)	1,312 (991)*	2,656 (1,434)

* 2 x combat radius



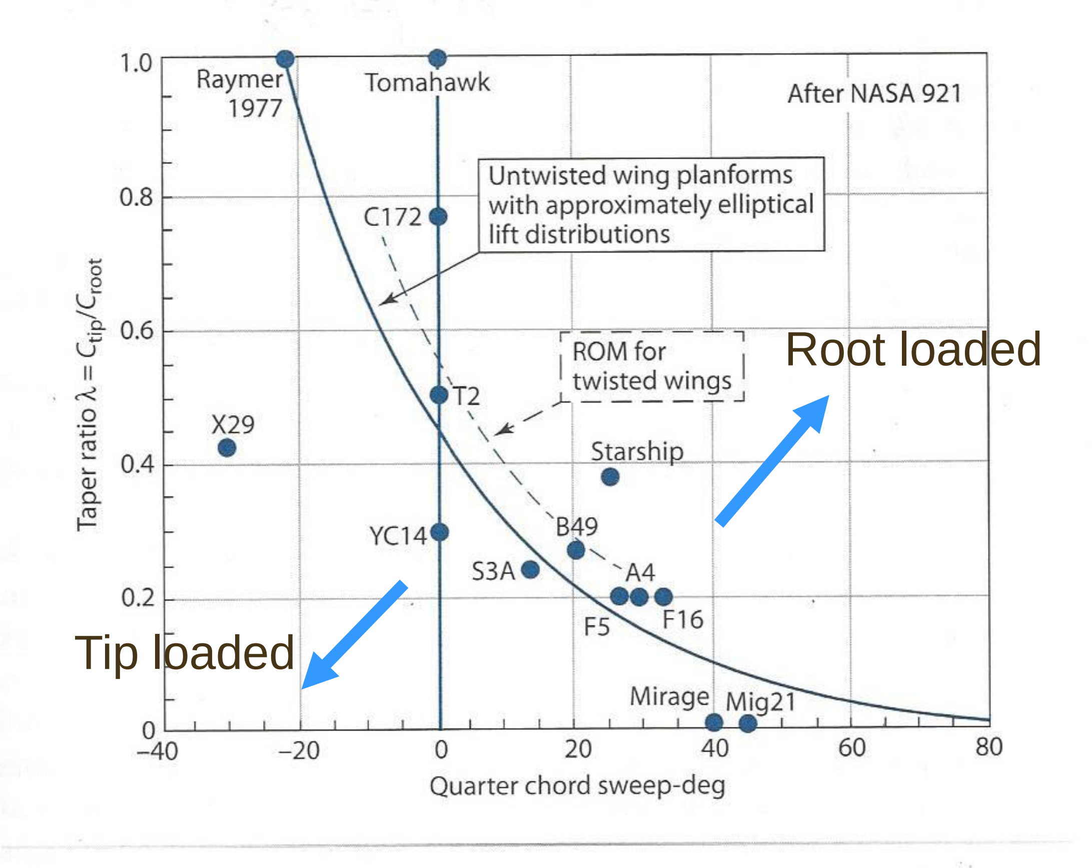
Schrenk's Approximation for Rectangular Planform

- Wing section aerodynamic load = (lift per unit span)/chord
- For an unswept, untwisted wing, lift distribution is represented by line midway between planform chord distribution and ellipse of equal area



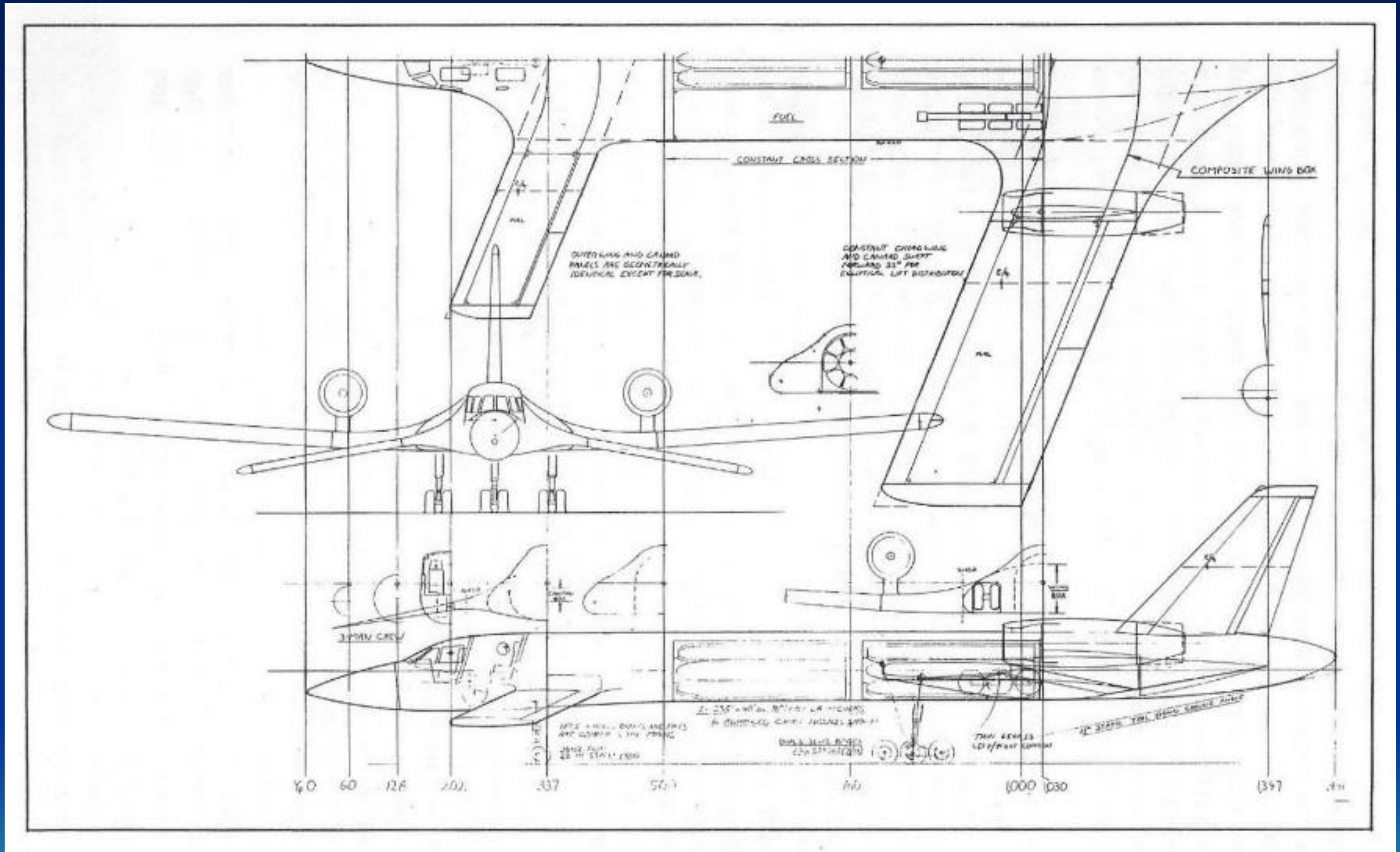
Downwash Effect is Accentuated

- Untapered, untwisted wing can have close to elliptical (minimum drag due to lift) lift distribution



Source: Raymer

If Wing is Swept Forward



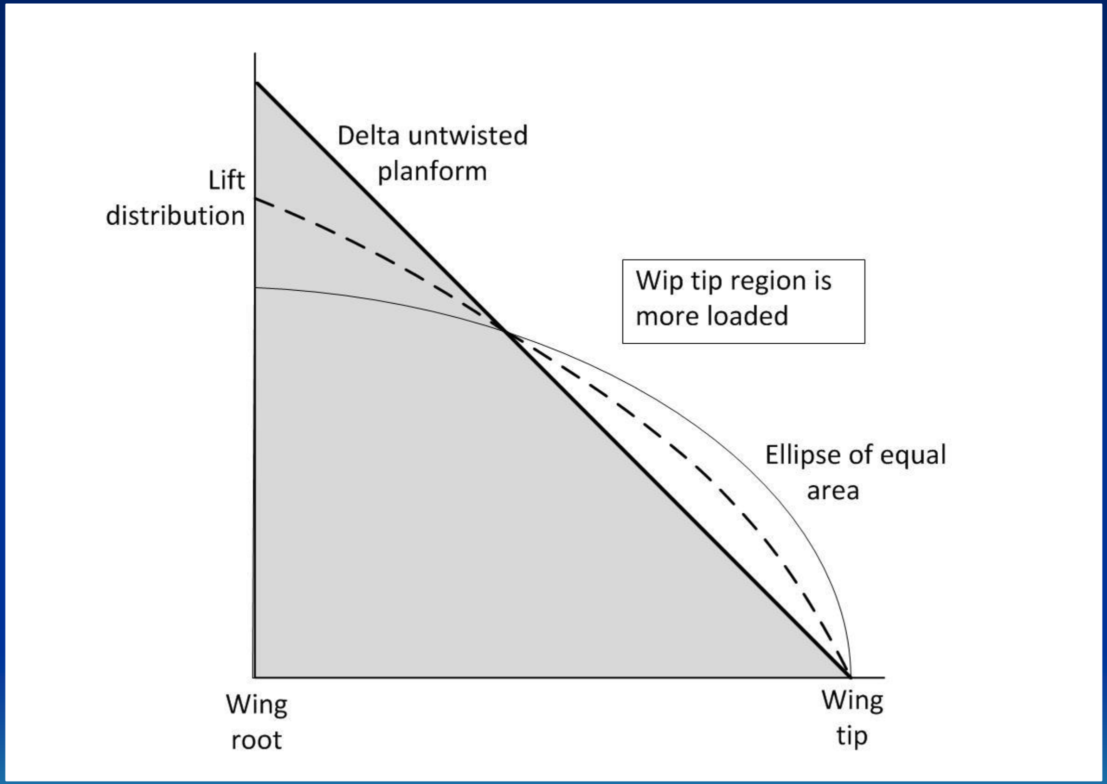
Source: Raymer

- Low-cost bomber concept

Forward swept wing unloads outboard wing sections even more, so that elliptic lift distribution can be achieved.

Schrenk's Rule for Delta Planform

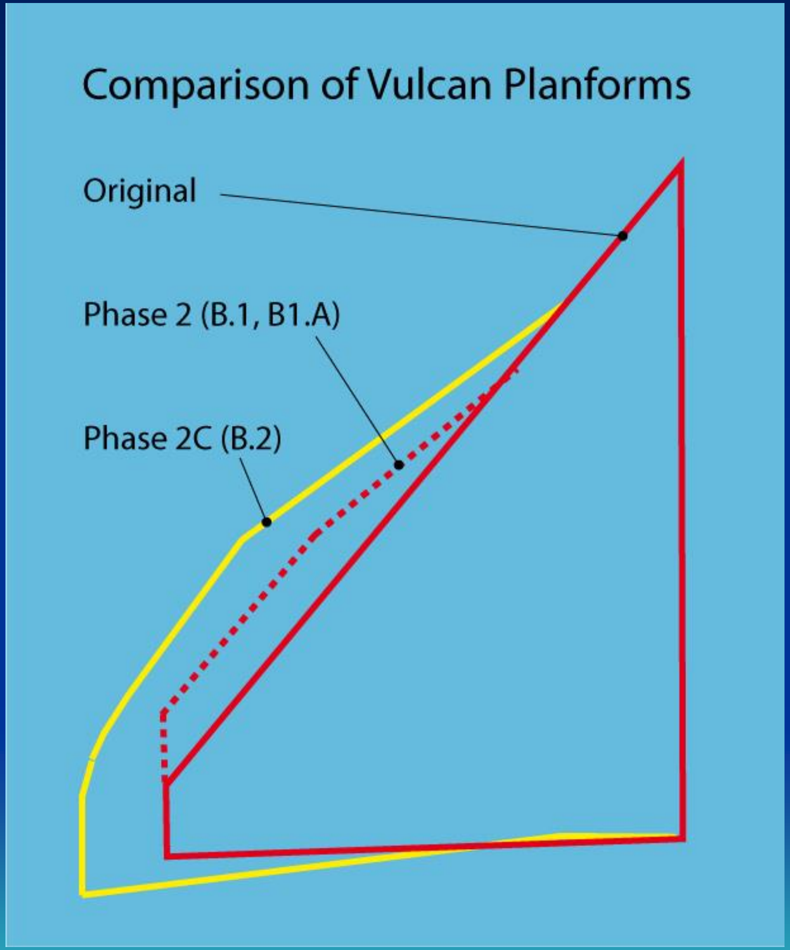
- Likelihood of asymmetric stall
- Increased transonic drag



Development of Avro Vulcan Planform

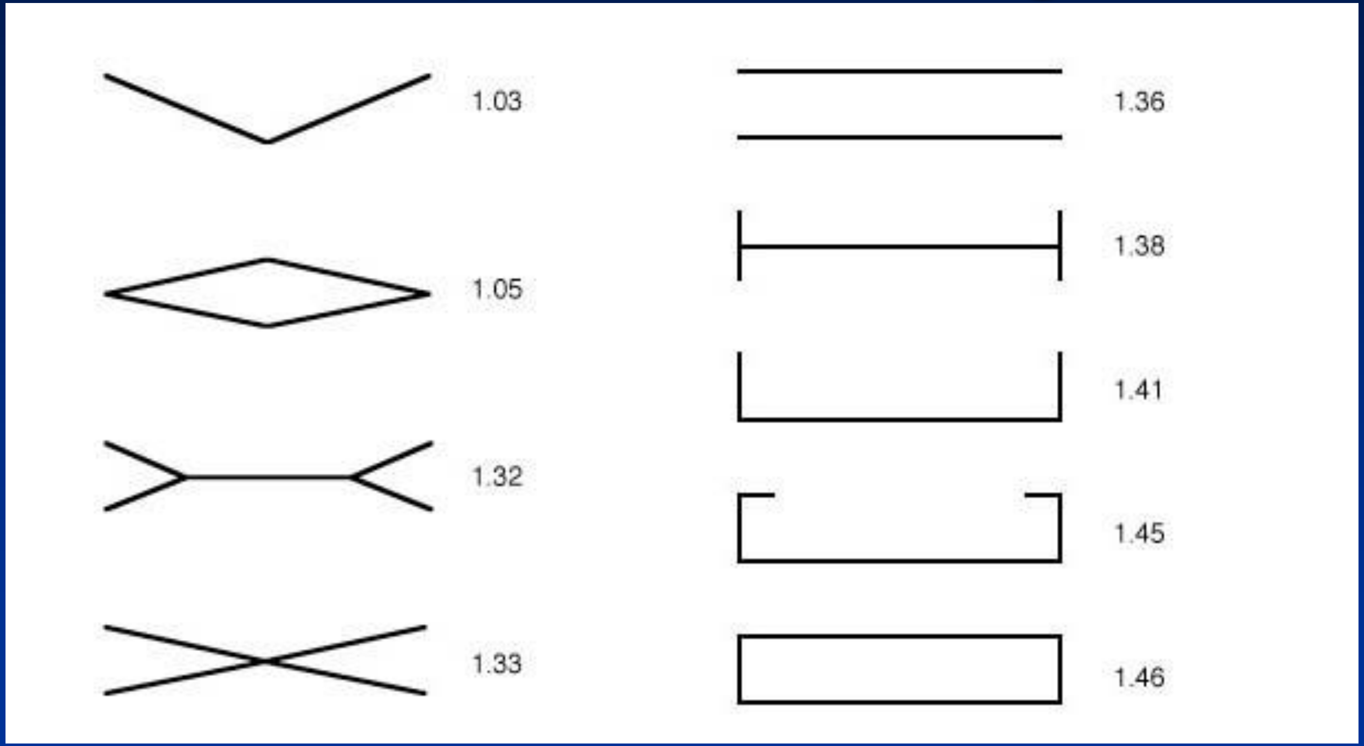
B.1

B.2



Source (all images): commons.wikipedia.org

Nonplanar Wings



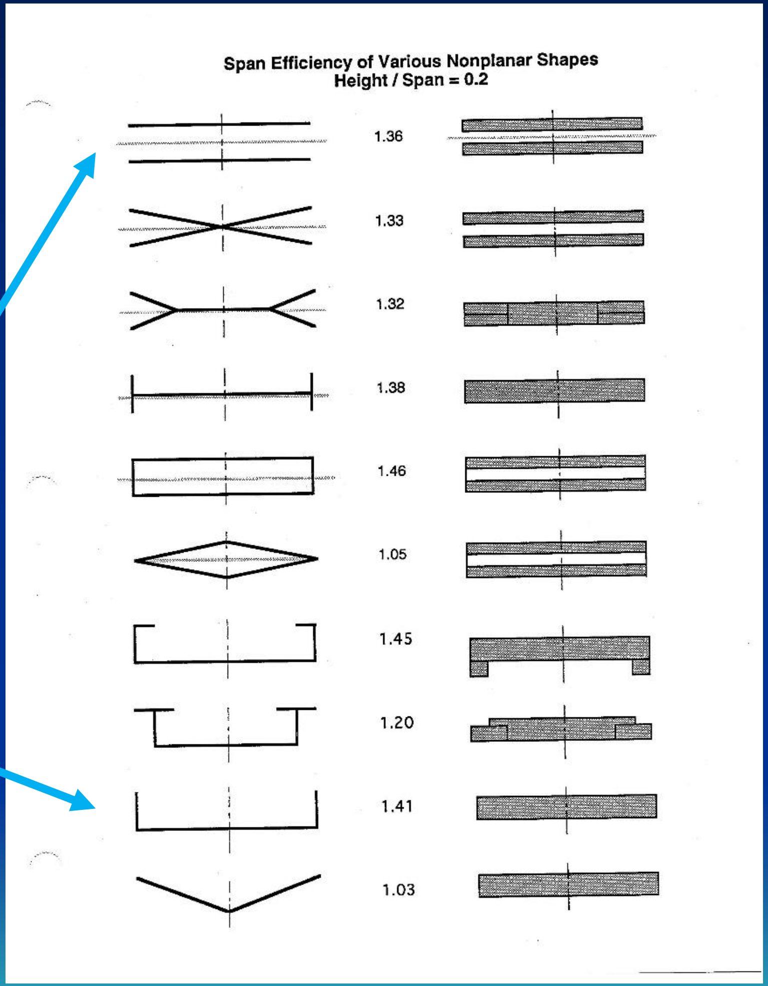
Dr. Ilan Kroo

Source: Kroo: Non-planar Wing Concepts for Increased Aircraft Efficiency

- Span efficiency of various optimally loaded non-planar wings (h/b = 0.2)
- Based on analysis by Prandtl

Non-planar Wing Planforms

- Span efficiency relative to rectangular wing of same *planform* area and *span*.
- Each biplane wing has 2X AR of single plane wing
- Vertical surfaces reduce drag (like winglets), but don't count in area



See John McMasters Collected Works on www.adac.aero

Source: John McMasters

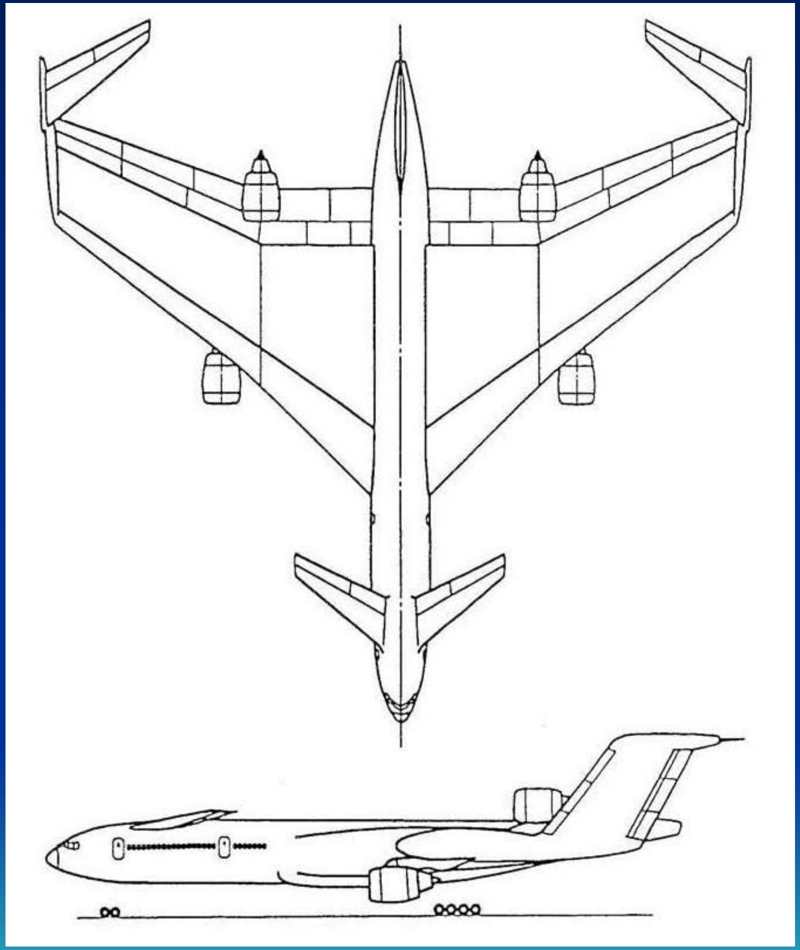
Box Wing

- Oswald efficiency factor 1.46
- FARs require longitudinal static stability
- MLG attached to fuselage
- Narrow chord wing has little structural depth
- Must also resist flexure from engine moments
- Where does fuel go?

Same planform area, but ½ volume

C-Wing

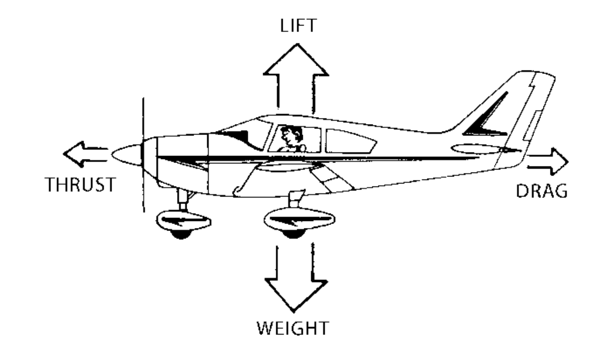
- McMasters/Kroo/Pavek concept
- Hybrid blended wing-body

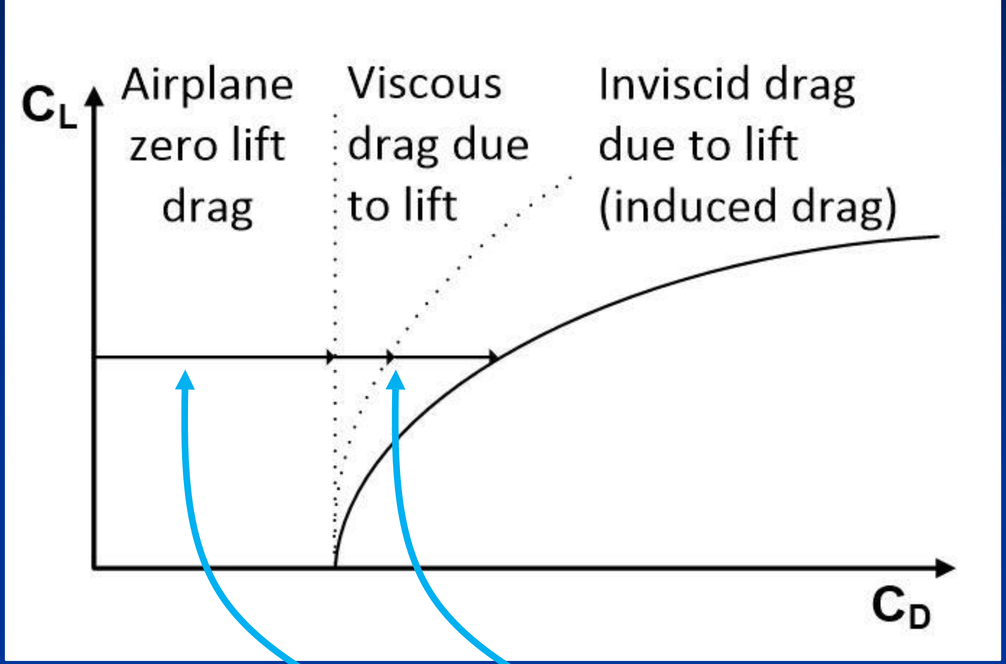


Source: John McMasters

Drag Polar

Flying at $(L/D)_{max}$, half the drag is directly dependent on weight





$$C_D = C_{D_0} + KC_L{}^2$$

where $C_L = \dfrac{L}{\frac{1}{2}\rho V^2 S}$

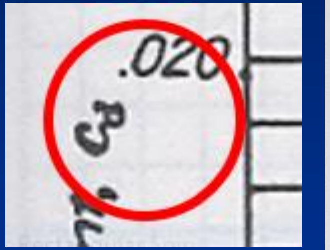
and $L = W$

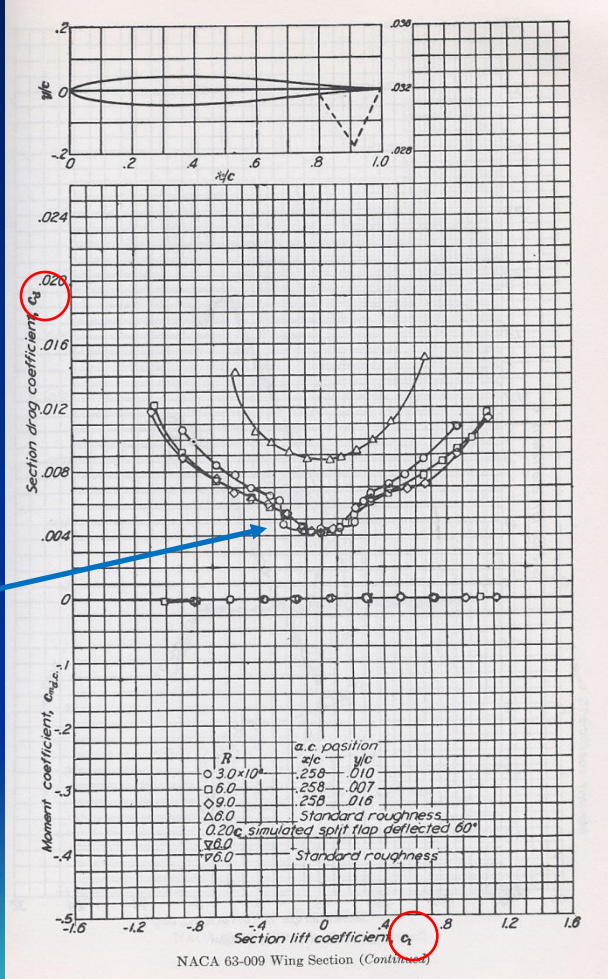
At $(L/D)_{max}$ condition $C_{D_0} = C_{D_i}$

C_{D_i} includes viscous drag due to lift

Example of Forces on a 2-D Airfoil

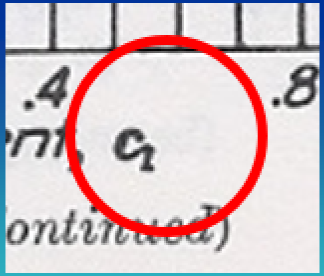
- Drag is primarily due to increased shear forces
- <u>No</u> induced drag
- But it <u>is</u> part of drag due to lift
- Note drag bucket near α = +/-2^0 due to laminar flow





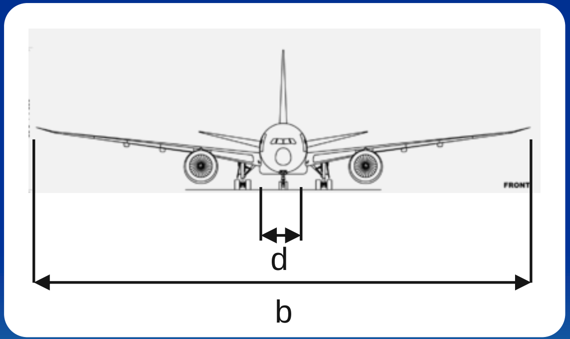
Source: Abbott & Von Doenhoff

Lower case suffixes imply <u>section</u> force coefficients

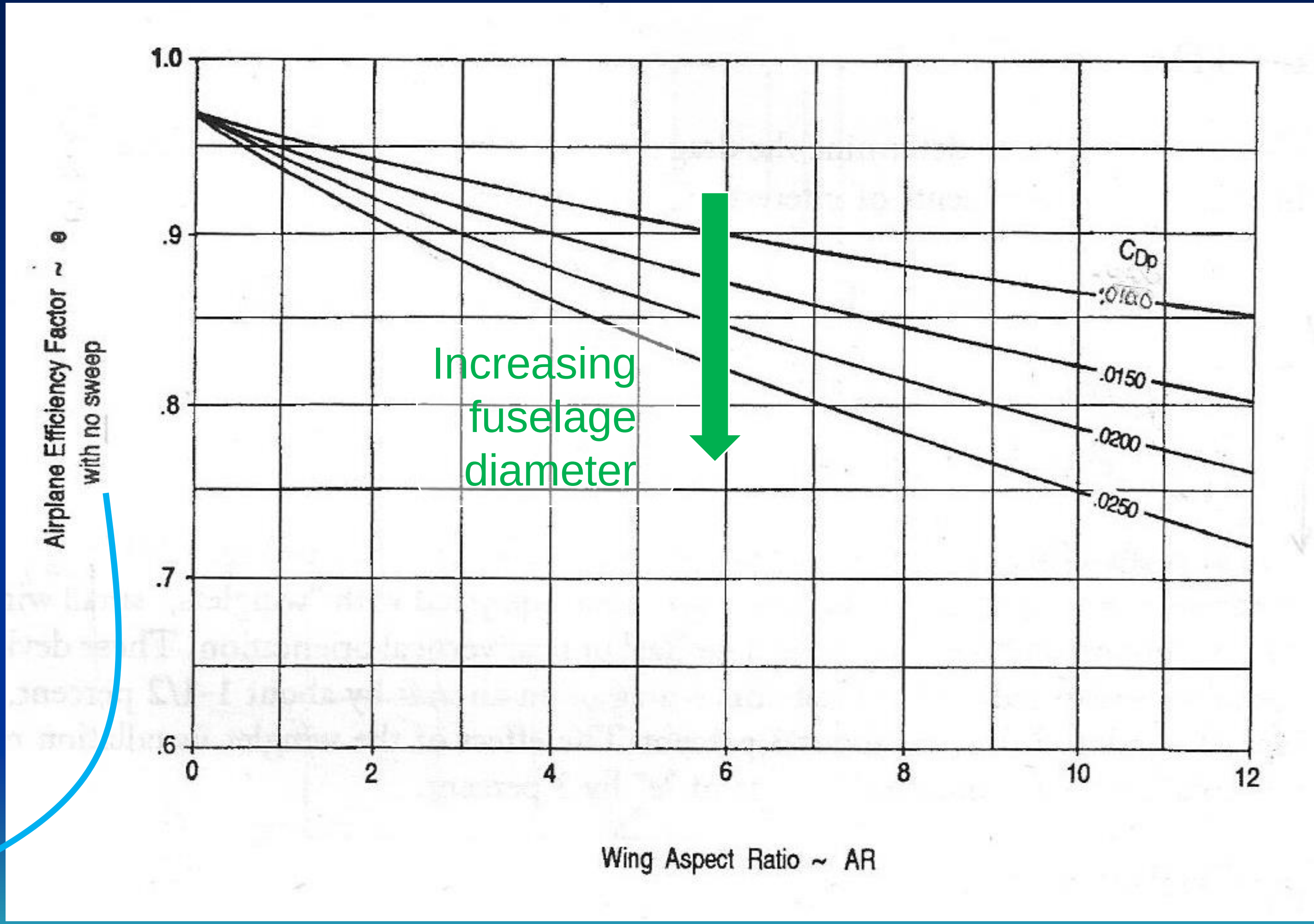


Oswald Efficiency Factor for Airliners (Shevell Method)

- Uses C_{D_P} (= C_{D_o}) as a surrogate for d_{fuse}/b
- As d_{fuse}/b increases, spanwise lift distribution is less elliptical



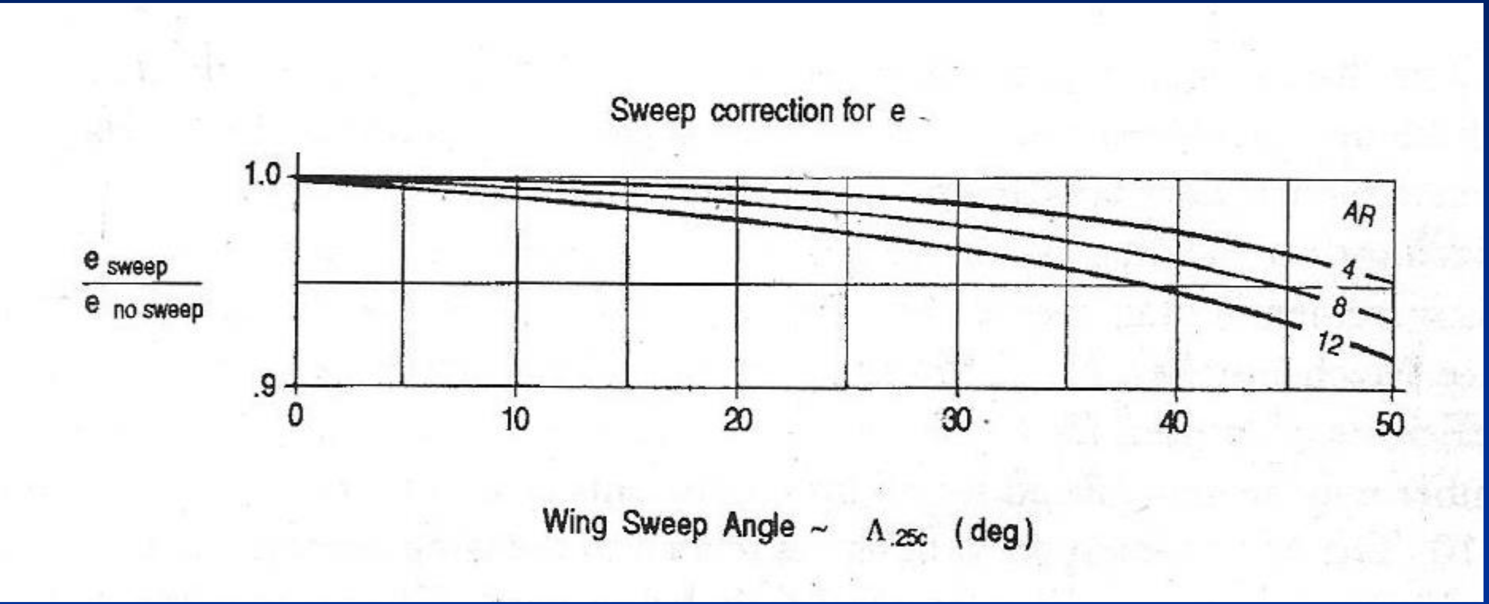
See next chart



Source: Schaufele

Oswald Efficiency Factor for Airliners (Shevell Method)

- Sweep correction factor for e



Source: Schaufele

Estimation of Oswald Efficiency Factor

Symbol is white circle

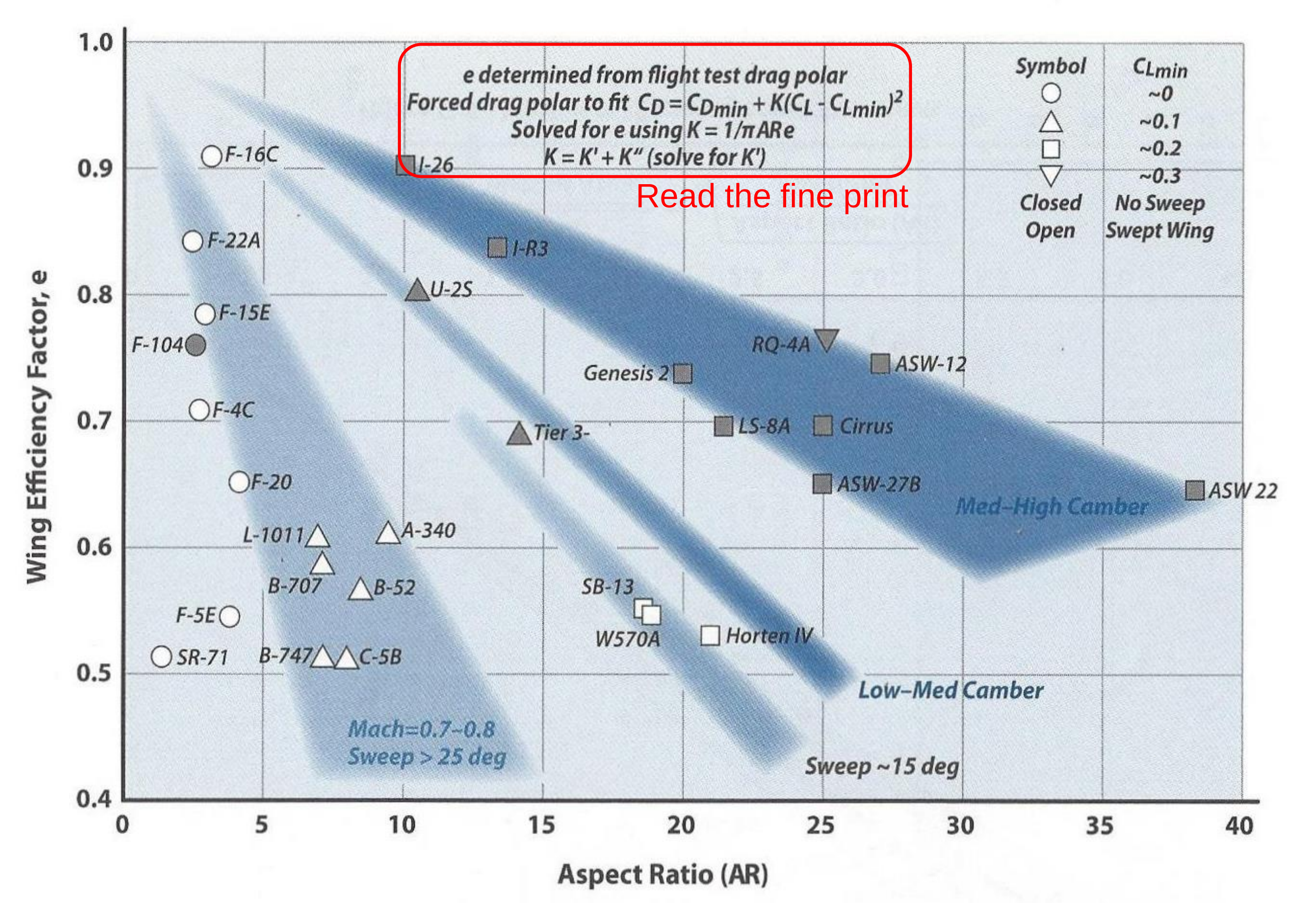
Except for condition $C_{L_{min}} = 0$

values of e shown here are not valid when used in equation

$$C_D = C_{D_0} + \frac{1}{\pi AR\, e} C_L^2$$

They are valid in

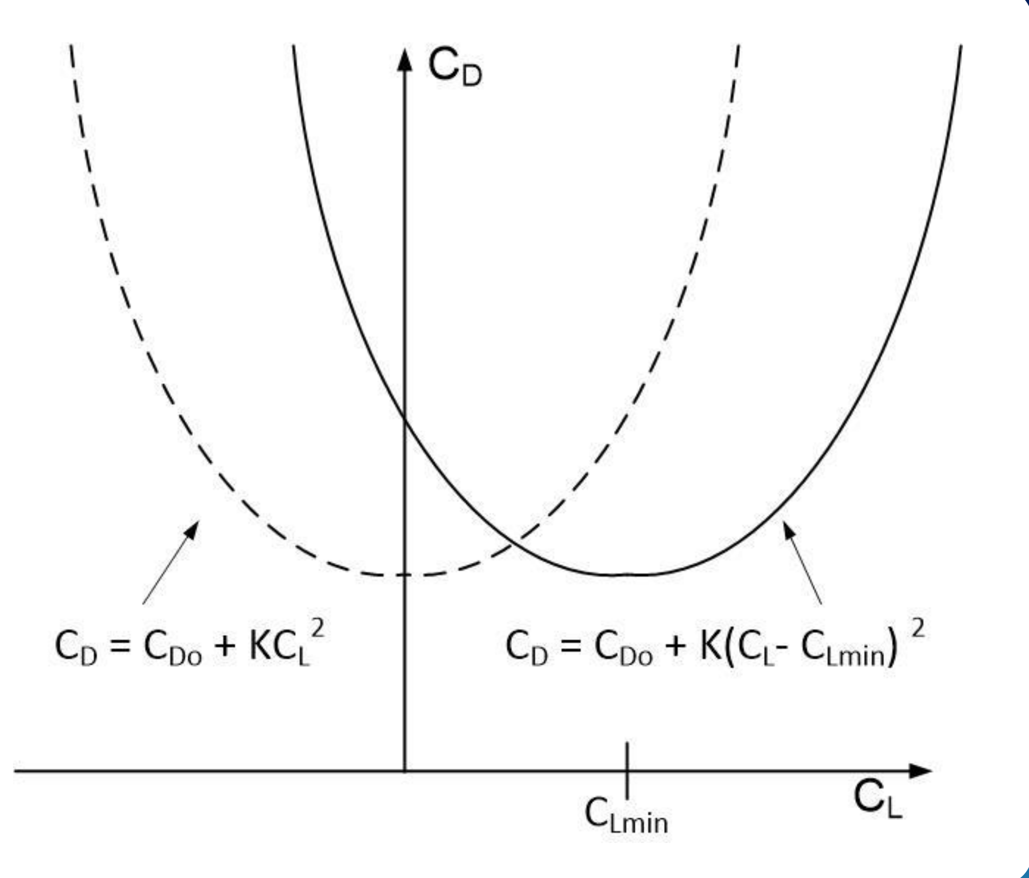
$$C_D = C_{D_{min}} + \frac{1}{\pi AR\, e}\left(C_L - C_{L_{min}}\right)^2$$



Source: Nicolai/Carichner

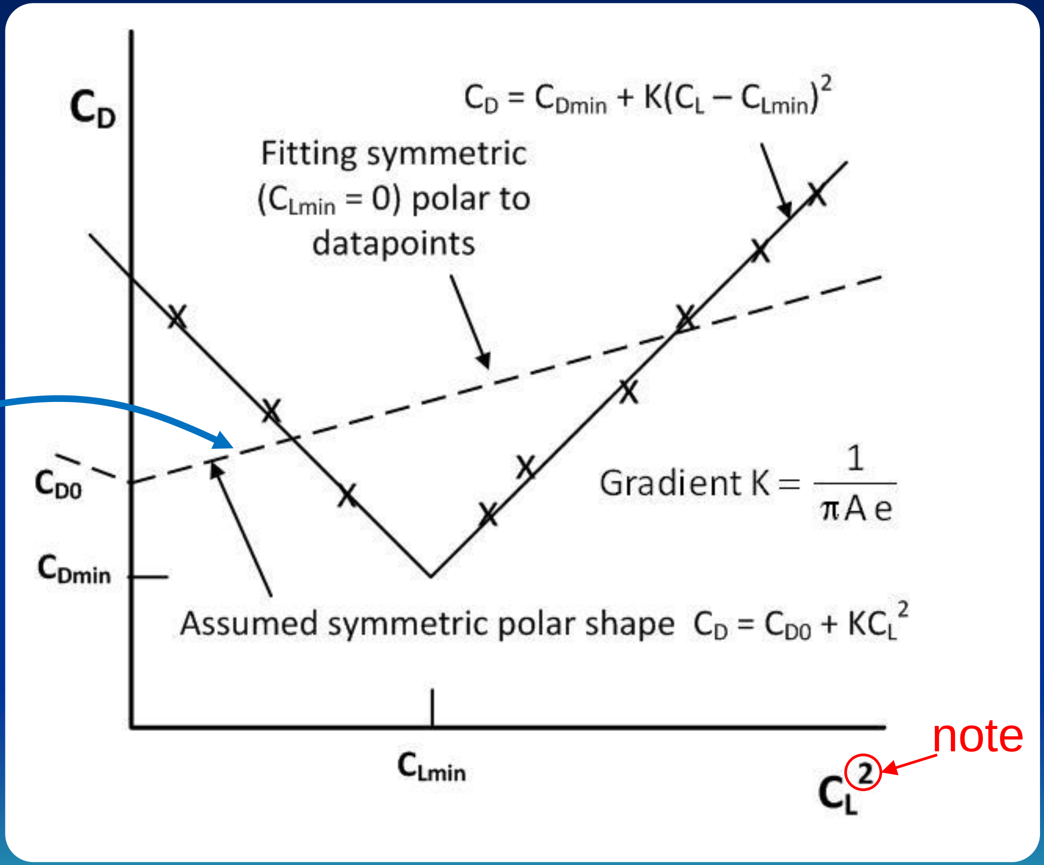
Drag Polar Comparison

- In Raymer's analysis, all polars are assumed symmetric ($C_D = C_{D_o} + K\,C_L^2$)
- In practice, except for aerobatic and fighter aircraft, polars are not symmetric



Caveat for Oswald Efficiency Factor Chart

- Values of e using Raymer analysis are only valid for $C_{L_{min}} = 0$ (white circles on previous chart)
- If symmetric polar is assumed, values of K are lower (e is higher)



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Ground Effect on K

Include in FAR 25.111 and 25.121(a) climb requirements for 1st segment (up to 35 ft AGL)

OGE, tip vortex flow is unconstrained

https://www.boldmethod.com/blog/lists/2017/02/5-factors-that-affect-vortex-strength/

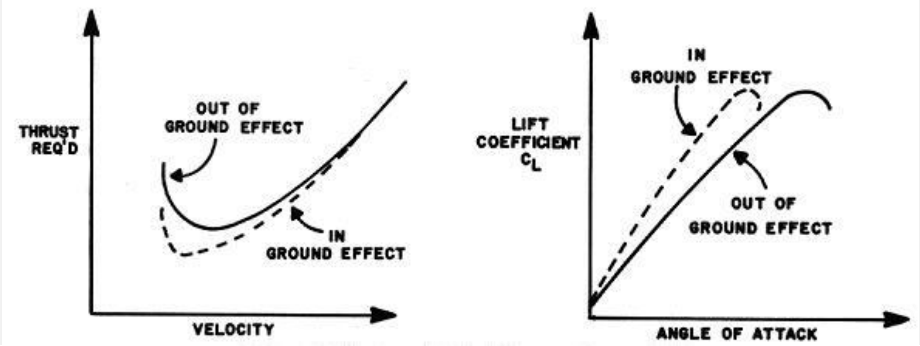
$$\frac{K_{effective}}{K} = \frac{33\left(\frac{h}{b}\right)^{1.5}}{1+33\left(\frac{h}{b}\right)^{1.5}}$$

Raymer Eq. 12.60

where

$h =$ height of wing above ground

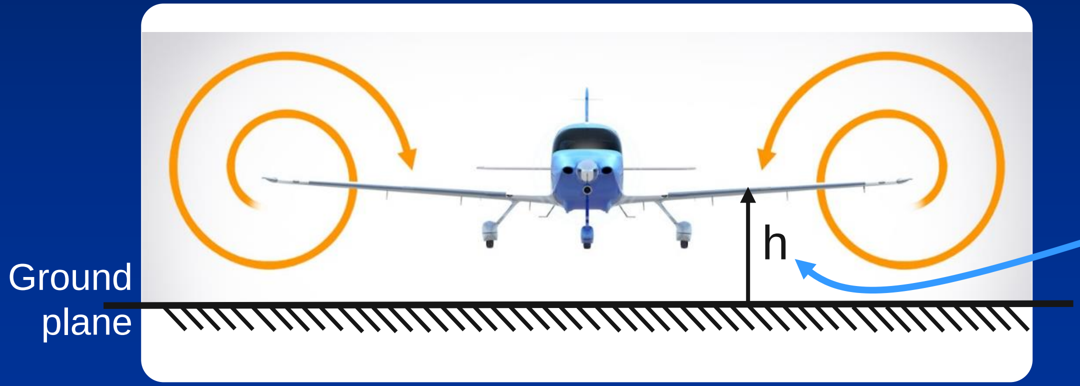
$b =$ wing span



http://www.faatest.com/books/FLT/Chapter17/GroundEffect.htm

Ground Effect on K

Include in FAR 25.111 and 25.121(a) climb requirements for 1st segment (up to 35 ft AGL)



https://www.boldmethod.com/blog/lists/2017/02/5-factors-that-affect-vortex-strength/

In potential flow, how do you meet the requirement for no flow through ground plane?

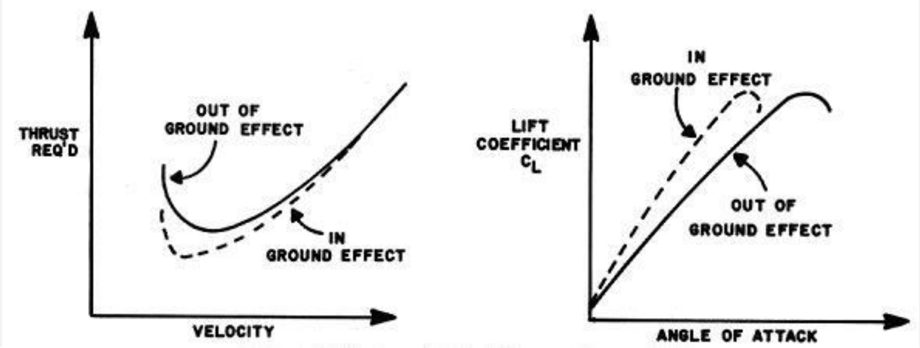
$$\frac{K_{effective}}{K} = \frac{33\left(\frac{h}{b}\right)^{1.5}}{1 + 33\left(\frac{h}{b}\right)^{1.5}}$$

Raymer Eq. 12.60

where

$h =$ height of wing above ground

$b =$ wing span



http://www.faatest.com/books/FLT/Chapter17/GroundEffect.htm

Ground Effect on K

Include in FAR 25.111 and 25.121(a) climb requirements for 1st segment (up to 35 ft AGL)

$$\frac{K_{effective}}{K} = \frac{33\left(\frac{h}{b}\right)^{1.5}}{1 + 33\left(\frac{h}{b}\right)^{1.5}}$$

Raymer Eq. 12.60

where

$h =$ height of wing above ground

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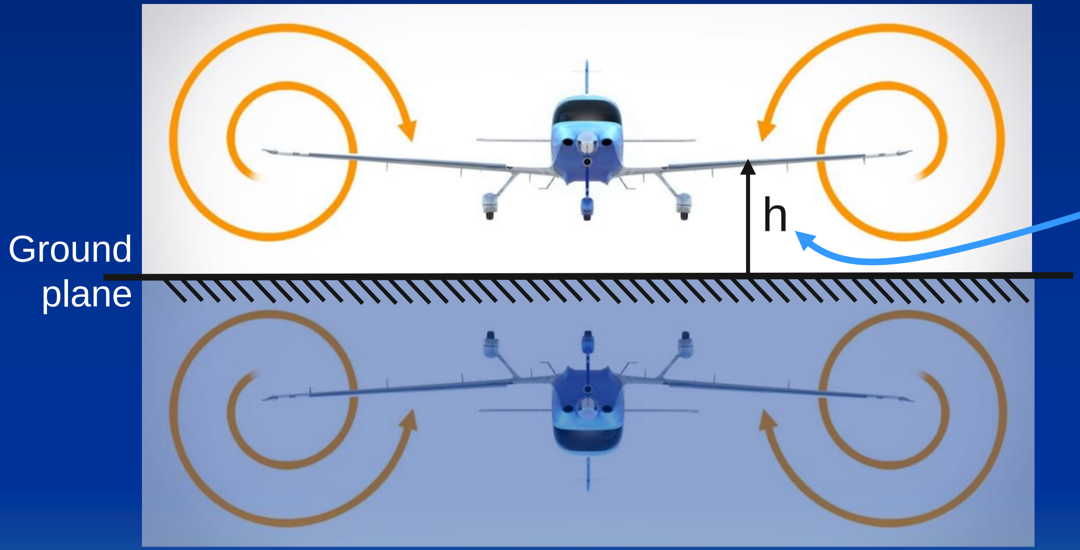
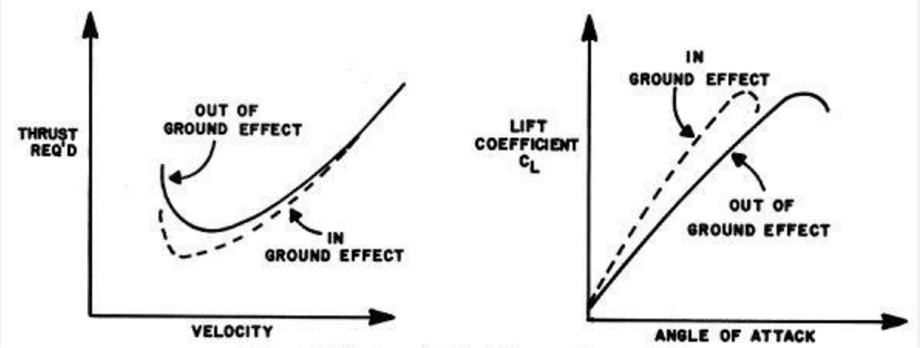


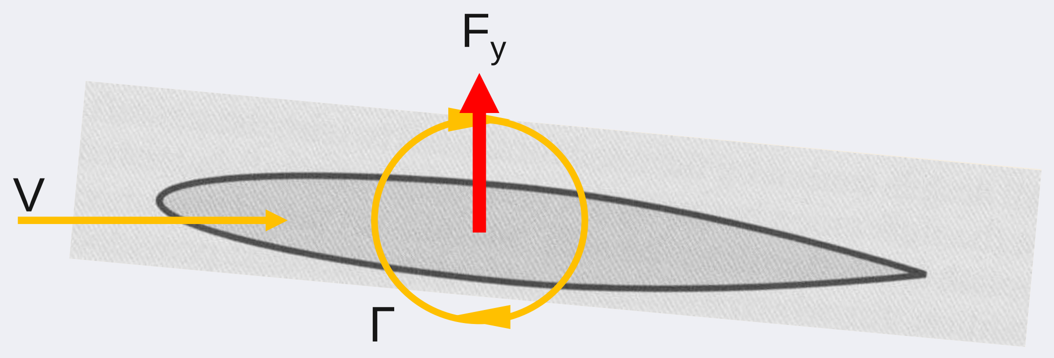
https://www.boldmethod.com/blog/lists/2017/02/5-factors-that-affect-vortex-strength/

In potential flow, must have mirror image to satisfy requirement for no flow through ground plane
Mirror vortices almost cancel tip vortices



http://www.faatest.com/books/FLT/Chapter17/GroundEffect.htm

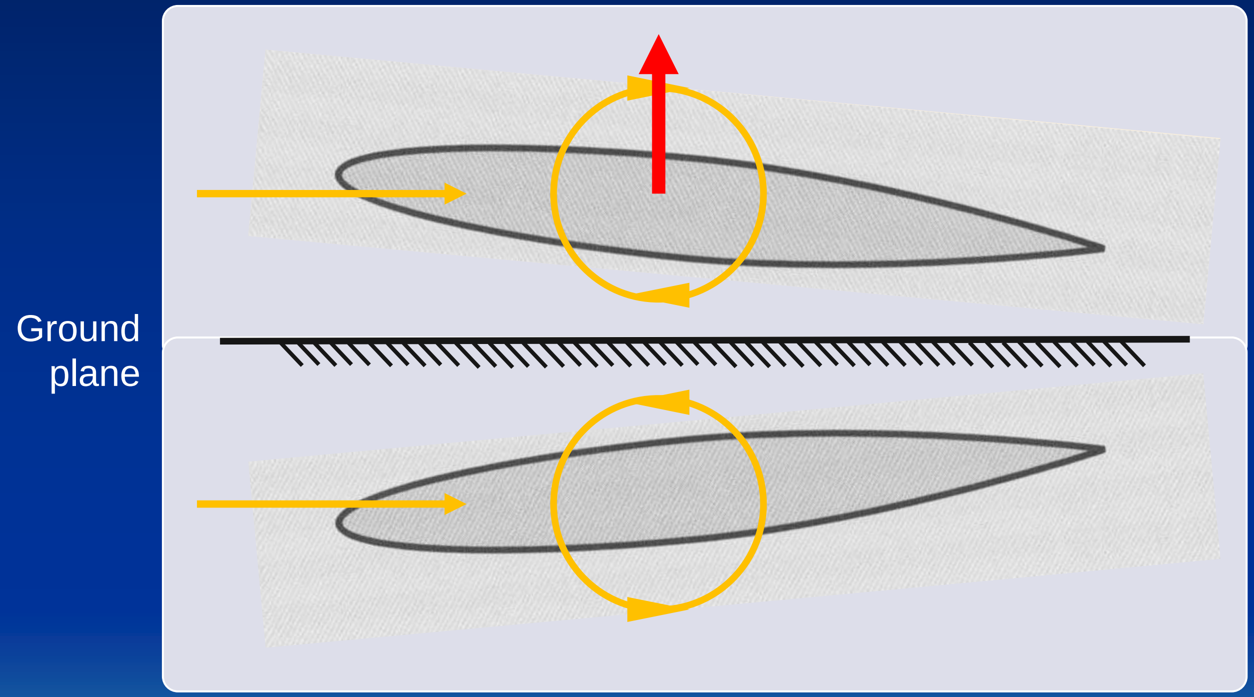
Effect of Ground Effect on Parasite Drag



$$F_y = \rho V \Gamma$$

In potential flow lift force results from interaction of uniform flow and bound vortex

Effect of Ground Effect on Parasite Drag



In potential flow, mirror vortex reduces velocity of flow in real flow, hence parasite drag

But effect is not usually considered in aircraft performance

Lift and High Lift Systems

Zero-Lift Drag C_{D_0}

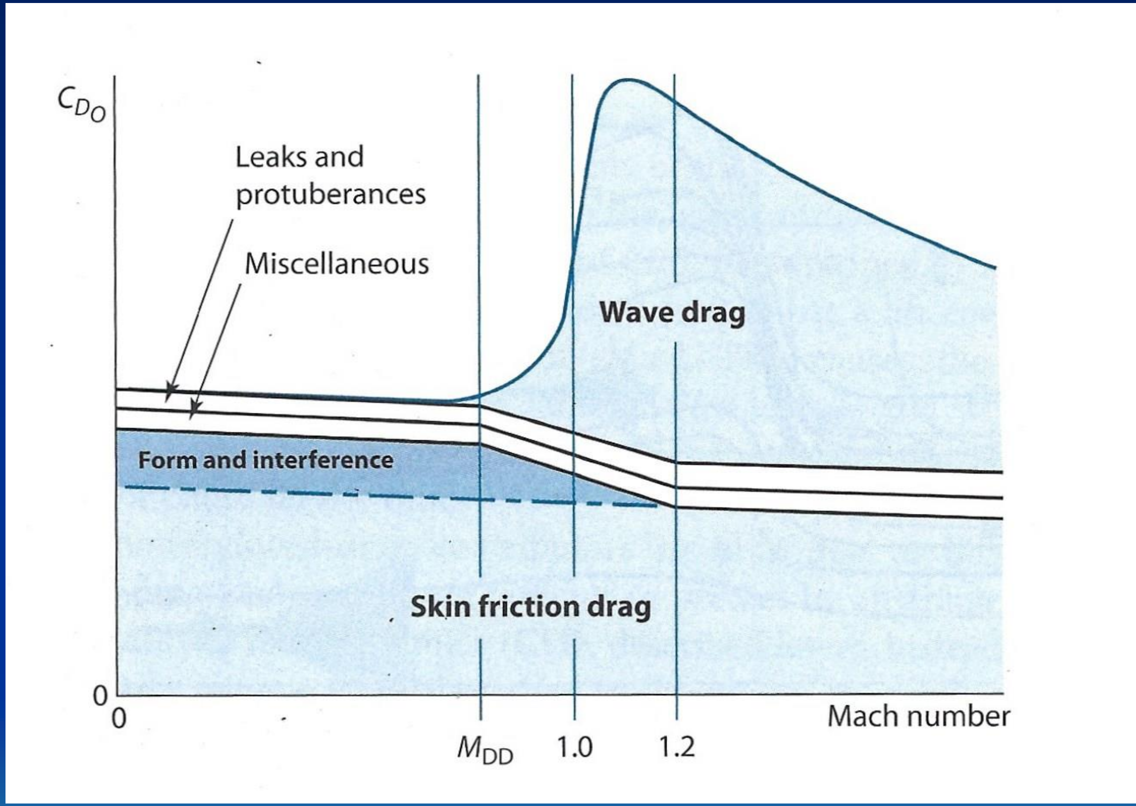
Drag due to Lift C_{D_i}

Wave Drag due to Volume $C_{D_{0_{supersonic}}}$

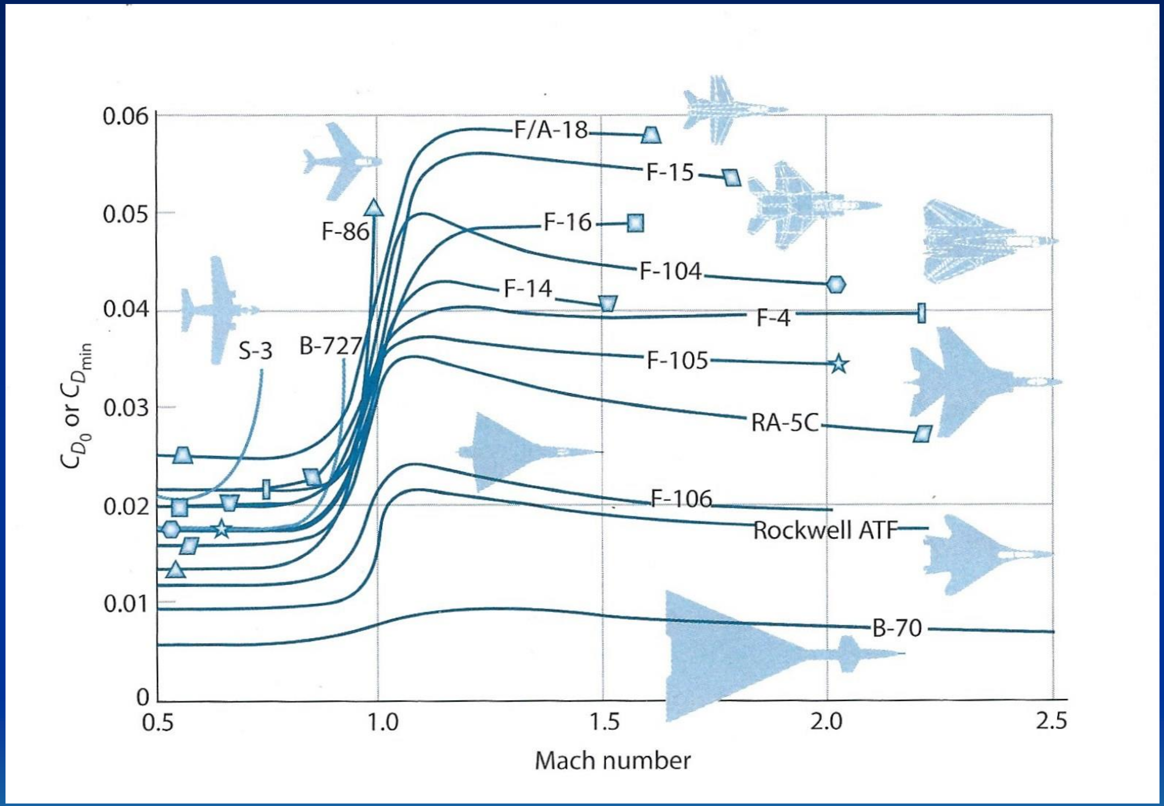
Wave Drag due to Lift C_{D_W}

Wing Design

Zero-Lift Wave Drag



© Raymer Fig. 12.33



© Raymer Fig. 12.34

ADAC Aircraft Design & Consulting

Sears-Haack Body

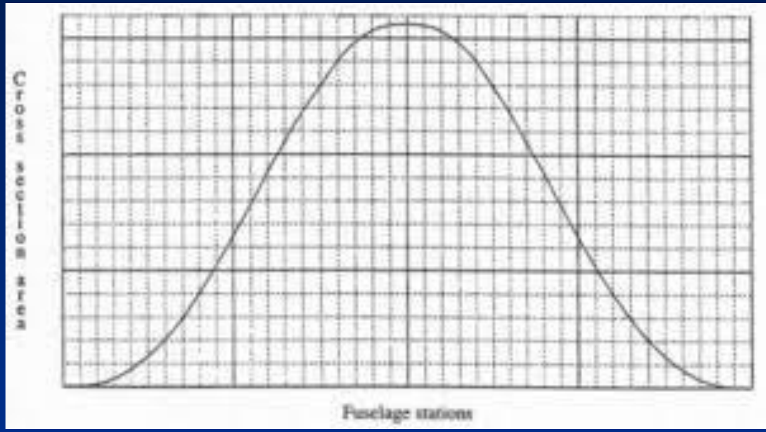
https://www.nae.edu/187408/WILLIAM-REES-SEARS-19132002

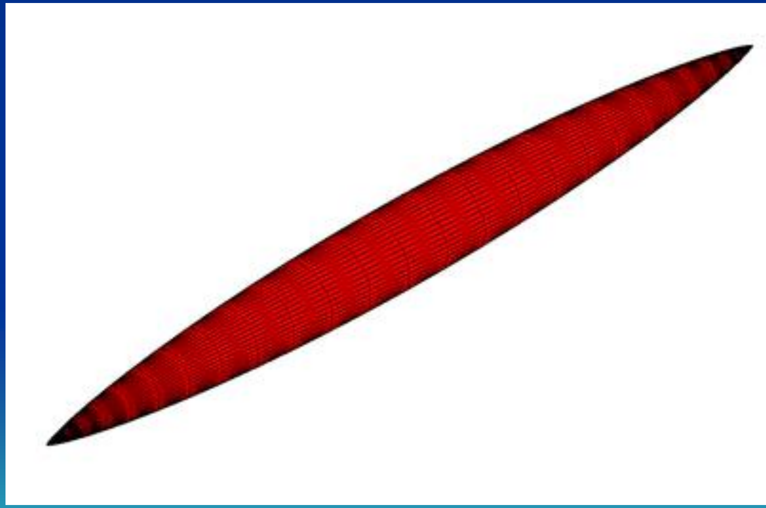
Bill Sears

- Minimum transonic wave drag for given volume
- For Sears-Haack body:

$$\left(\frac{D}{q}\right)_{wave} = \left(\frac{9\pi}{2}\right)\left(\frac{A_{max}}{l}\right)^2$$

where A_{max} = max x/s area
l = overall length

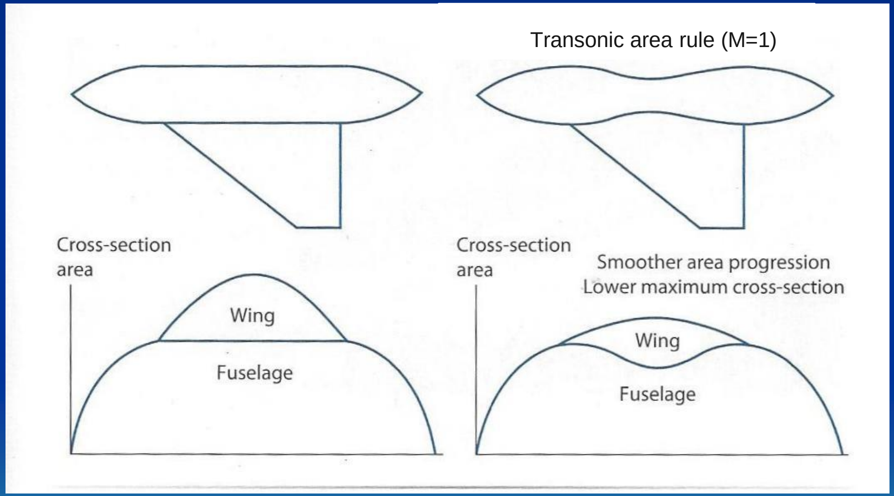




Area Ruling

YF-102

Area Rule developed by Richard Whitcomb at NASA Langley

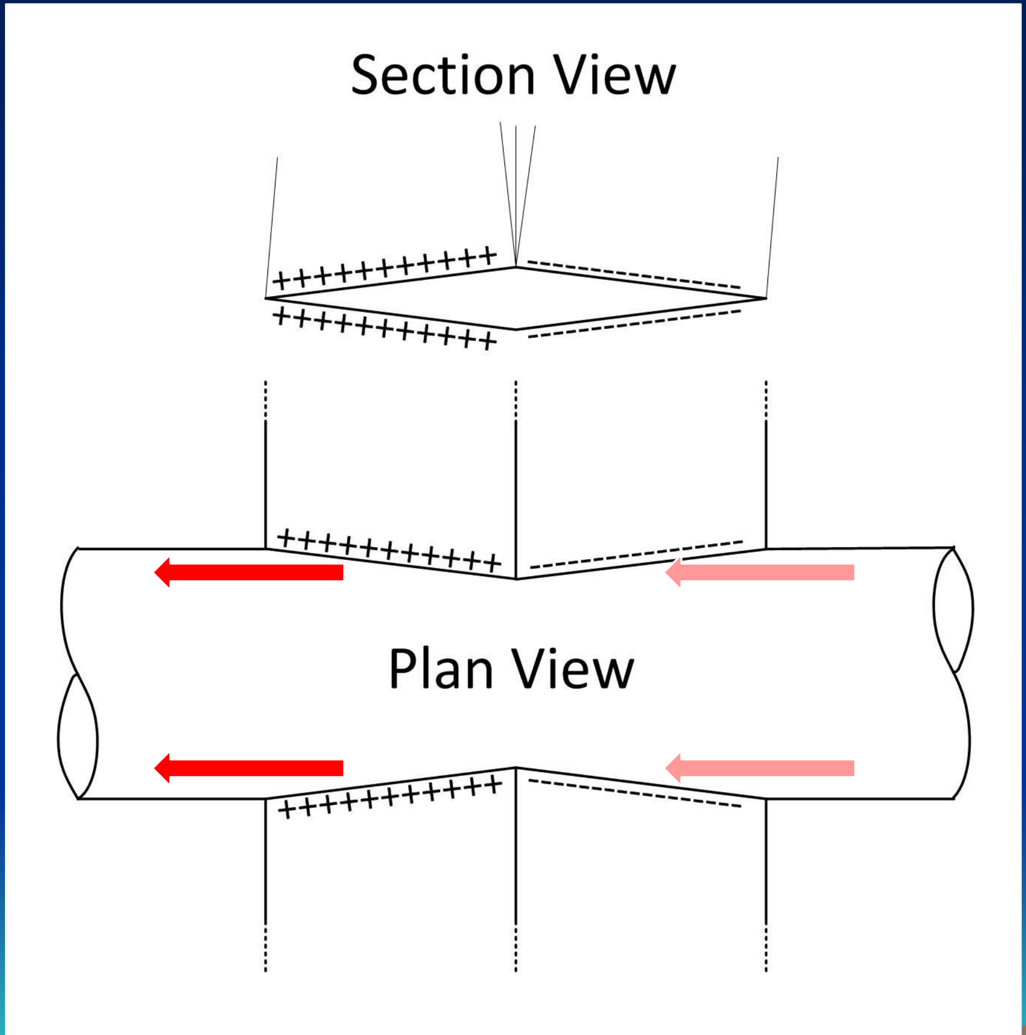


YF-102A

Transonic Area Ruling Simplified

Positive pressure on forward-facing wing surface increases drag

Positive pressure on aft-facing area of fuselage reduces drag

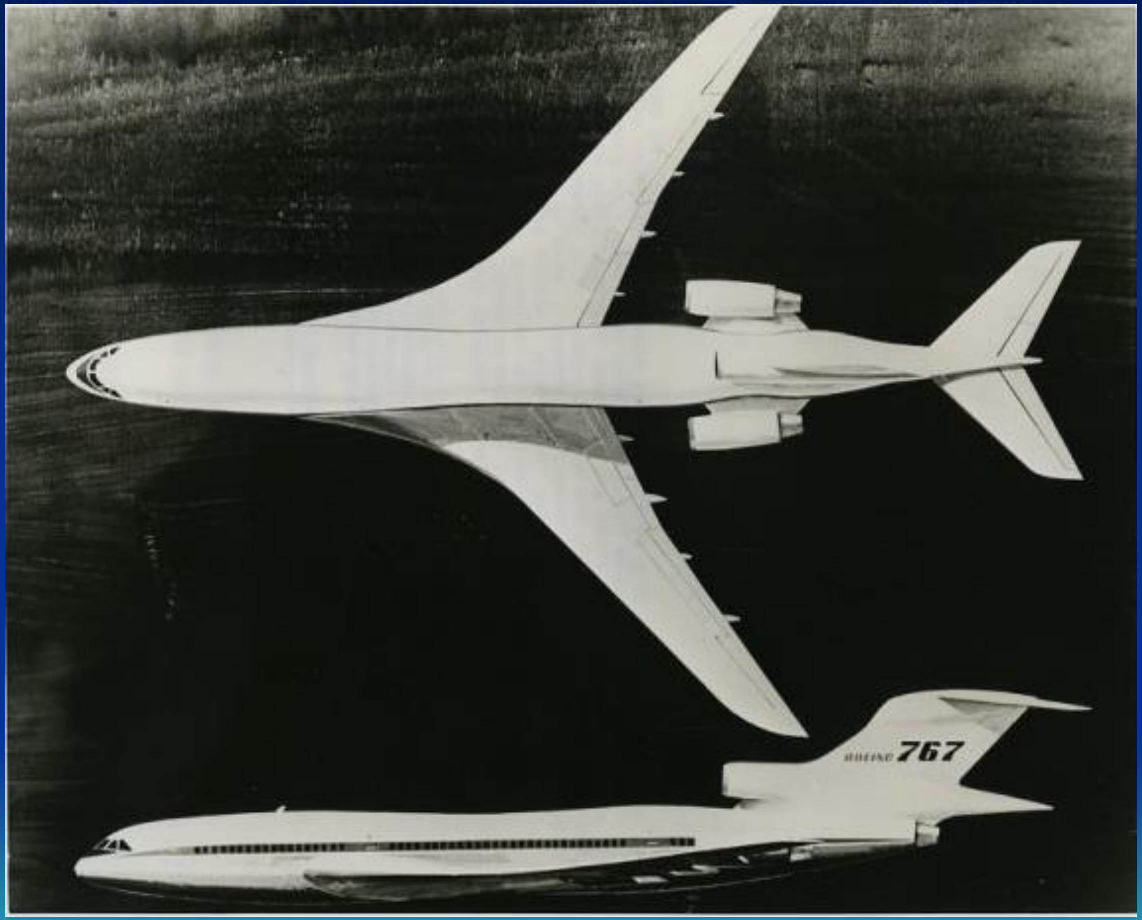


Negative pressure on aft-facing wing surface increases drag

Negative pressure on forward-facing area of fuselage reduces drag

Boeing Transonic Airliner

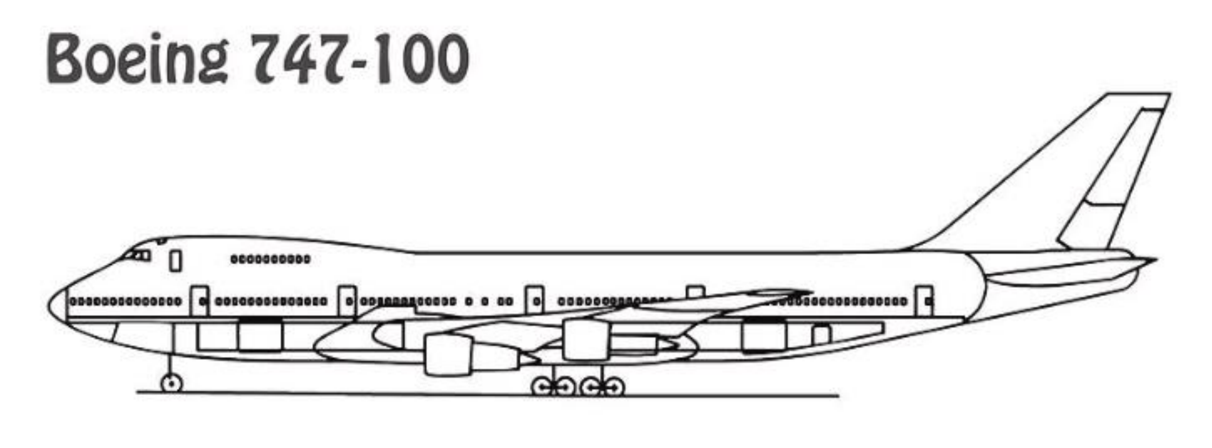
- Difficult and expensive to manufacture
- Inefficient seating
- Small reduction in flight time
- Small gain in aircraft and crew utilization
- Small gain in M L/D



Source: http://www.aerospaceprojectsreview.com/blog/?cat=9&paged=4

Area Ruling 747-200 vs -400

OML of extended upper cabin smoothed out area distribution and reduced zero-lift transonic drag



Source: pixels.com

https://magazin.lufthansa.com/xx/en/fleet/boeing-747-400-en/icon-of-the-airways/

Supersonic Parasite Drag

Raymer Eq. 12.42

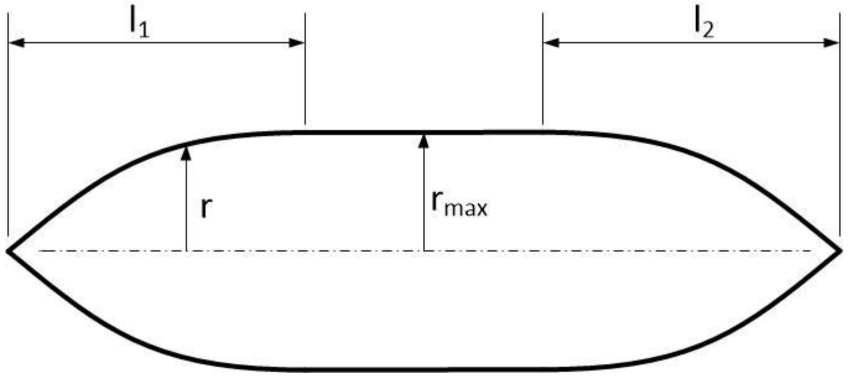
$$\frac{r}{r_{max}} = \left[1 - \left(\frac{x}{\frac{l}{2}}\right)^2\right]^{0.75}$$

Raymer Eq. 12.43

$$\text{for} -\frac{l}{2} \le x \le \frac{l}{2}$$

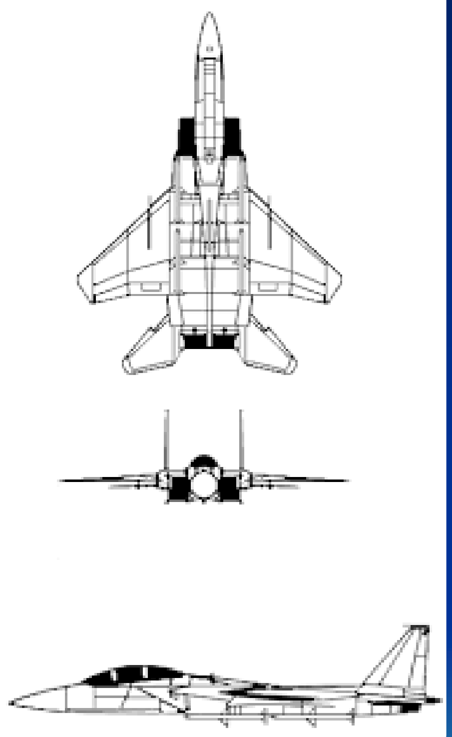
Raymer Eq. 12.44

$$\left(\frac{D}{q}\right)_{wave} = \frac{9\pi}{2}\left(\frac{A_{max}}{l}\right)^2$$



$l = l_1 + l_2$

Supersonic Parasite Drag



Λ_{LE} in degrees

$$\left(\frac{D}{q}\right)_{wave} = E_{WD}\left[1 - 0.386\,(M - 1.2)^{0.57}\left(1 - \frac{\pi\Lambda_{LE}^{0.77}}{100}\right)\right]\left(\frac{D}{q}\right)_{Sears\text{-}Haack}$$

where

$E_{WD} =$ empirical wave drag efficiency factor

For blended wing delta	$E_{WD} \approx 1.2$
For supersonic fighter, bomber or SST	$E_{WD} \approx 1.8 - 2.2$
For bumpy volume distribution	$E_{WD} \approx 2.5 - 3.0$
(F-15 optimized for dogfight)	$E_{WD} \approx 2.9$

Lift and High Lift Systems

Zero-Lift Drag C_{D0}

Drag due to Lift C_{Di}

Wave Drag due to Volume $C_{D0_{supersonic}}$

Wave Drag due to Lift C_{DW}

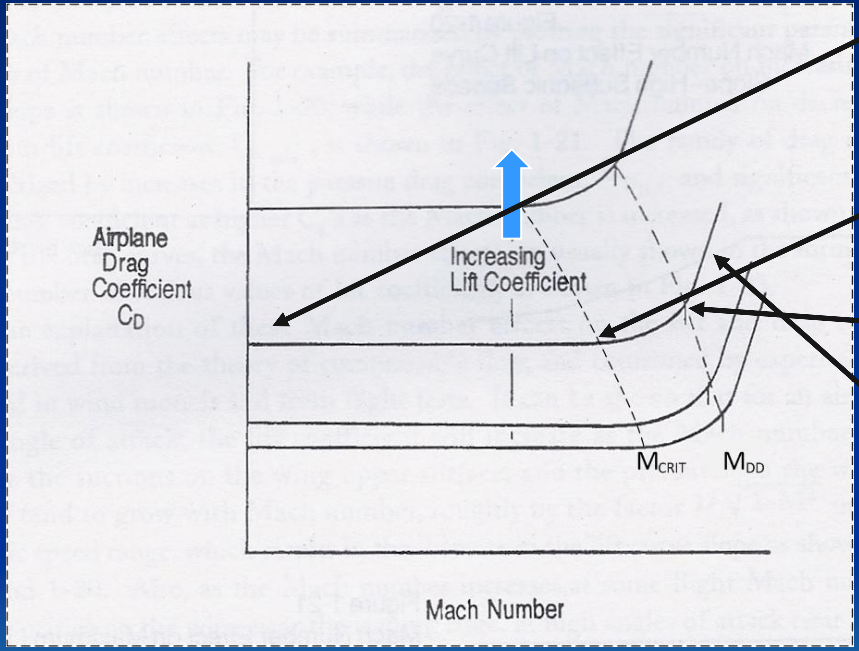
Wave Drag due to Lift C_{D_W}

Subsonic/Transonic

Supersonic

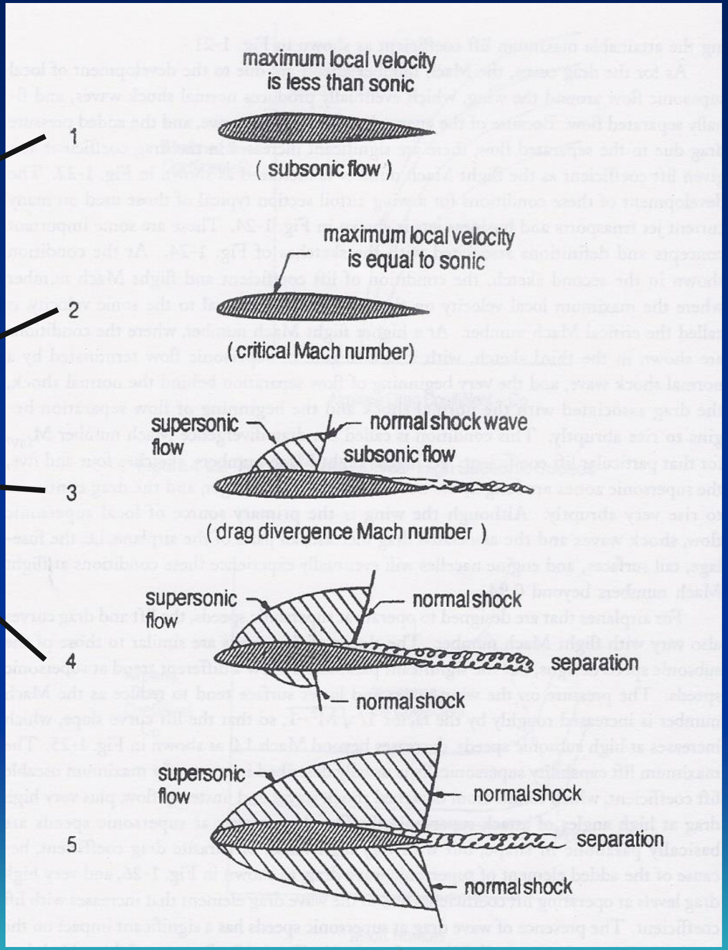
Flow Over Wing At Increasing Mach Number

M_{CRIT} and M_{DD} are a function of C_L (shown here), Λ and t/c



Source: Schaufele (modified)

Note: this is not a supercritical airfoil section



Source: Schaufele

Critical Mach No.

Drag Divergence Mach No.

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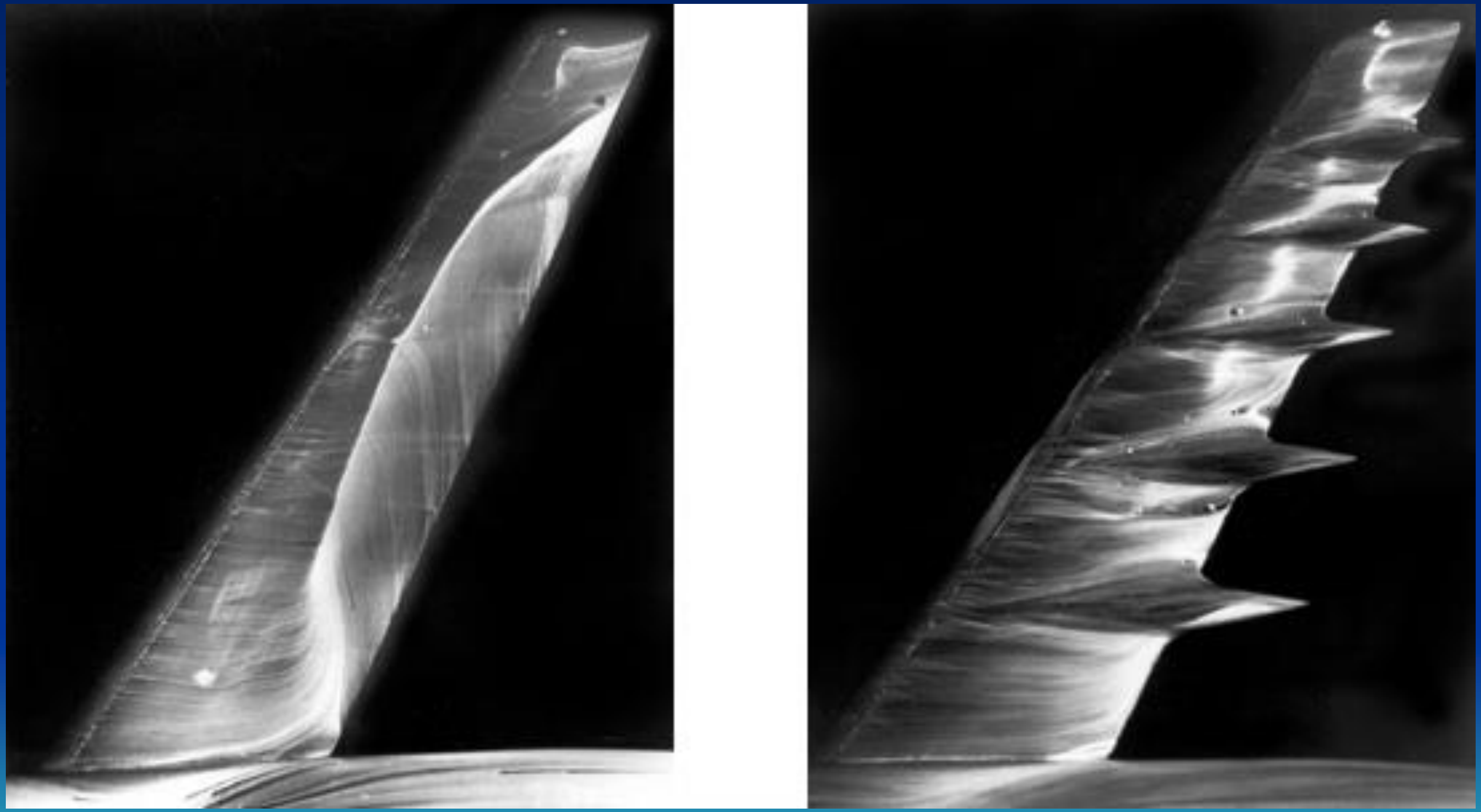
Anti-shock Bodies Eliminate Wing Shock

- Also called Whitcomb fairings or Küchemann carrots
- Led to development of supercritical airfoil sections

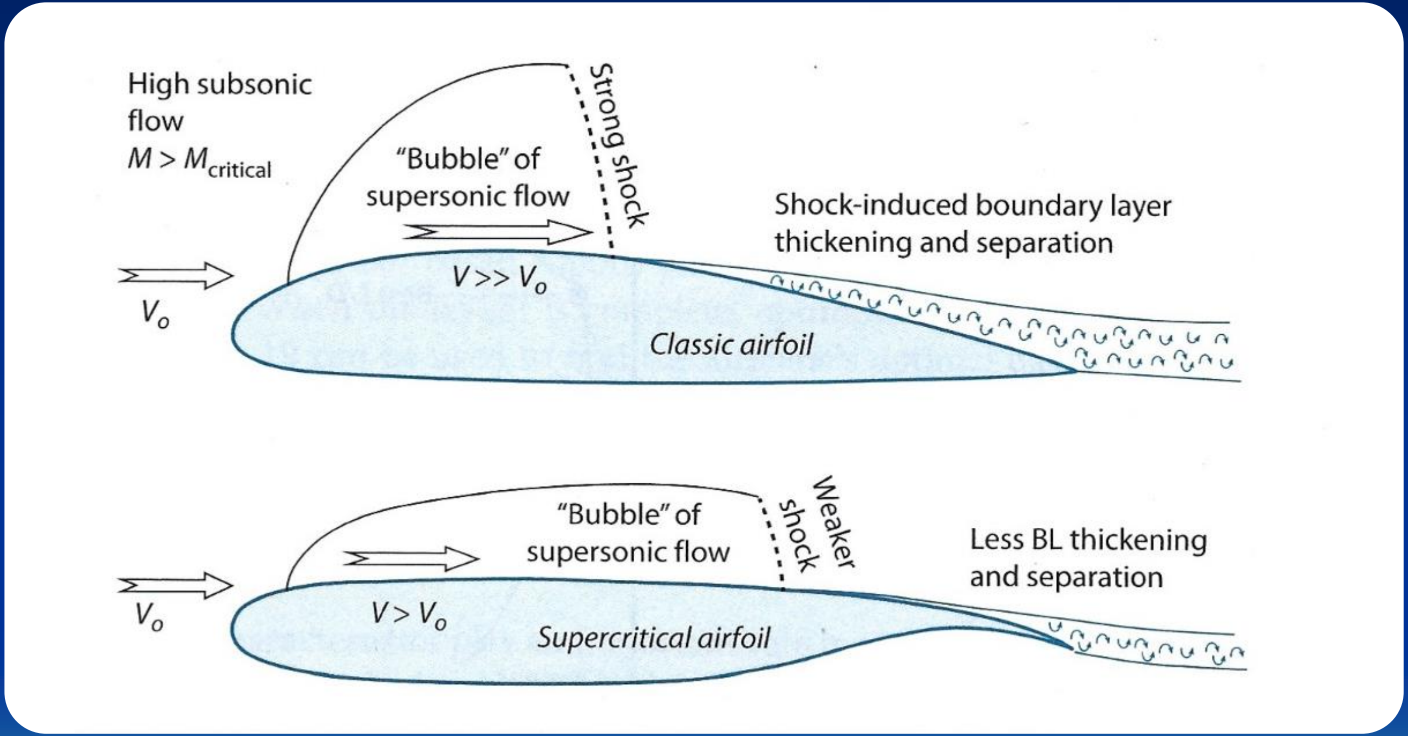


Küchemann Carrots on Convair 990

- Competed with B707 and DC-8
- First flight: January 1961
- Production run: 37

Conventional and Supercritical Airfoils

- Proposed in Germany in early 1940s
- Developed at Hawker Siddeley Hatfield in 1959-65, and by Richard Whitcomb in 1960s
- Supercritical airfoil reduces shock strength on upper surface
- Produces more uniform chordwise lift distribution



Raymer Fig. 4.8

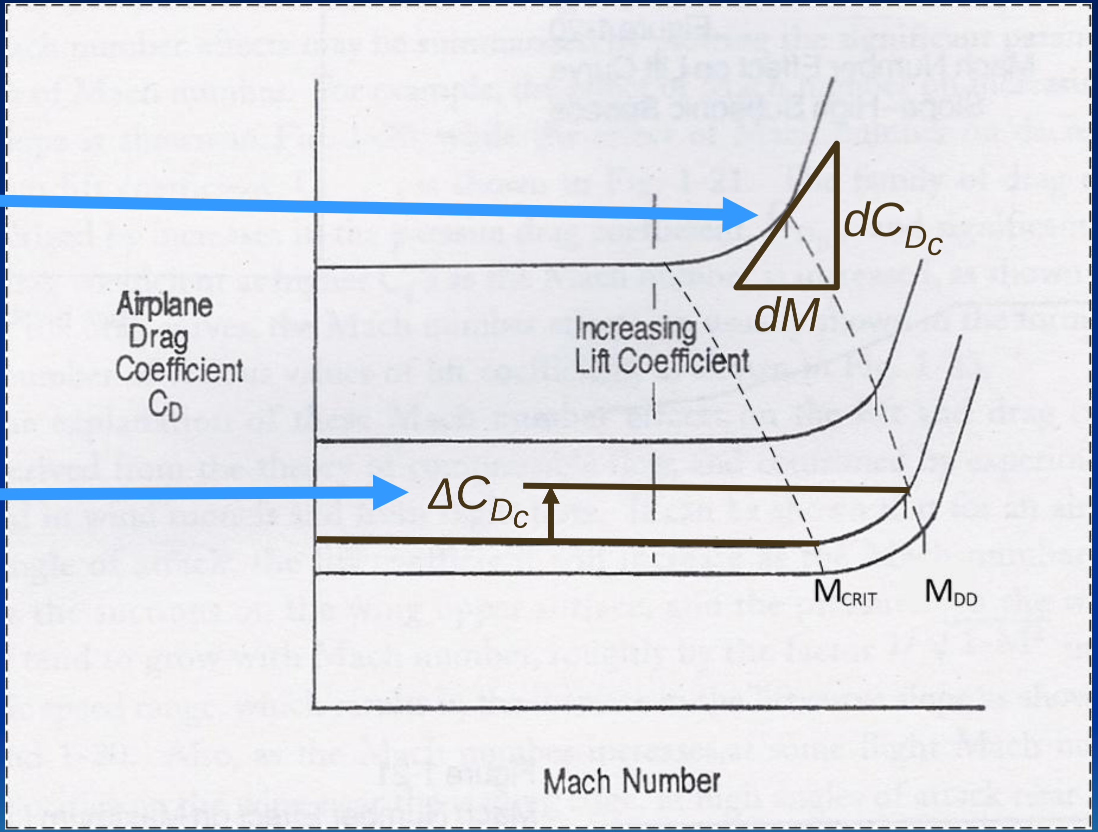
Definitions of Drag Divergence Mach Number

Douglas definition:

$$\frac{dC_{D_c}}{dM} = 0.10$$

Boeing definition:

$$\Delta C_{D_c} = 0.0020$$



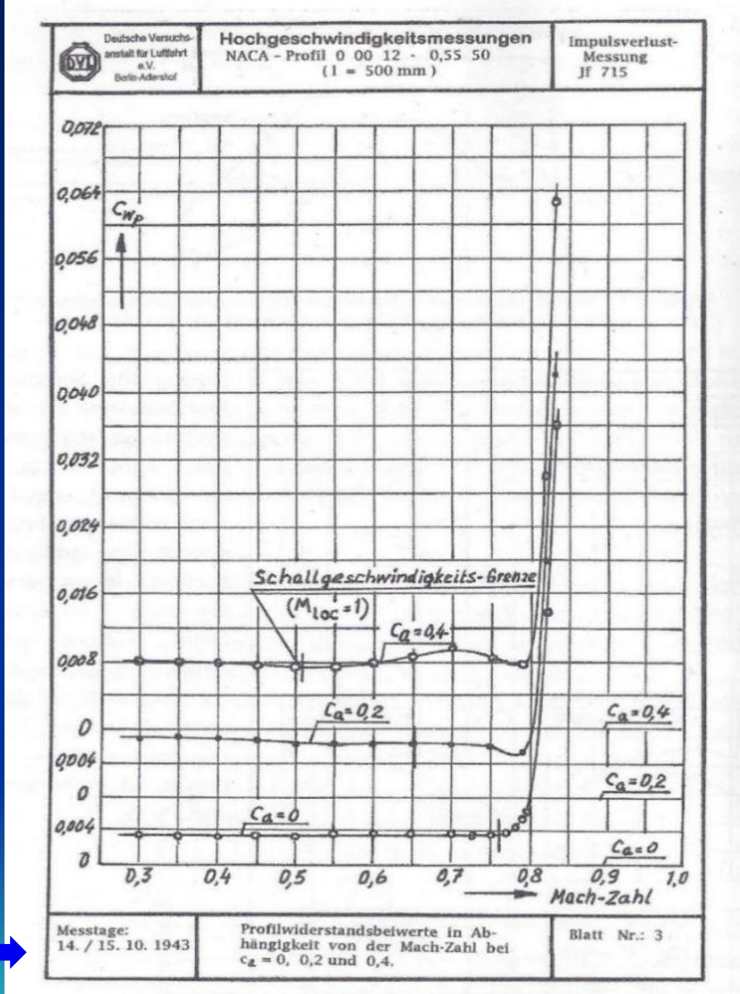
Source: Schaufele (modified)

Early Drag Map

Drag map for airfoil section NACA 0012.

Original source: UM 1167 (1944), B. Göthert

14/15 Oct 1943



Source: Obert

Me 262

$M_{MO} = 0.84$

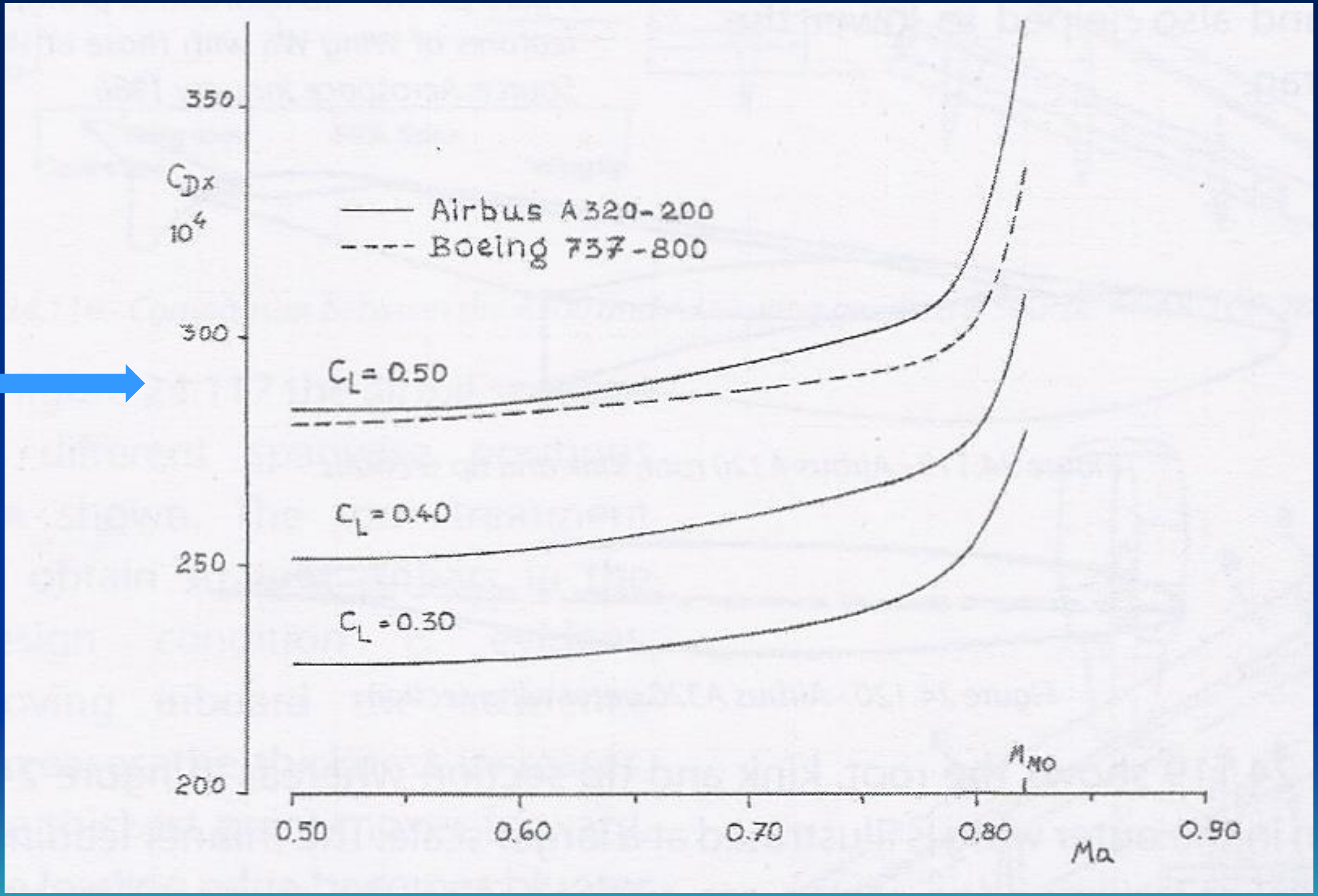
Wing sweep of 18.5^0 to balance heavier engines

First jet-powered flight 1942.07.18

Source: Wikpedia © Entity999

Textbooks containing Drag Plots

- Obert "Aerodynamic Design of Transport Aircraft" 2009
  - Many examples of commercial aircraft drag plots
- Schaufele "The Elements of Aircraft Preliminary Design"
- Shevell "Fundamentals of Flight" 1989
  - DC-10 C_L vs. C_D, L/D and ML/D (as fn. of C_L and M)



Source: Obert

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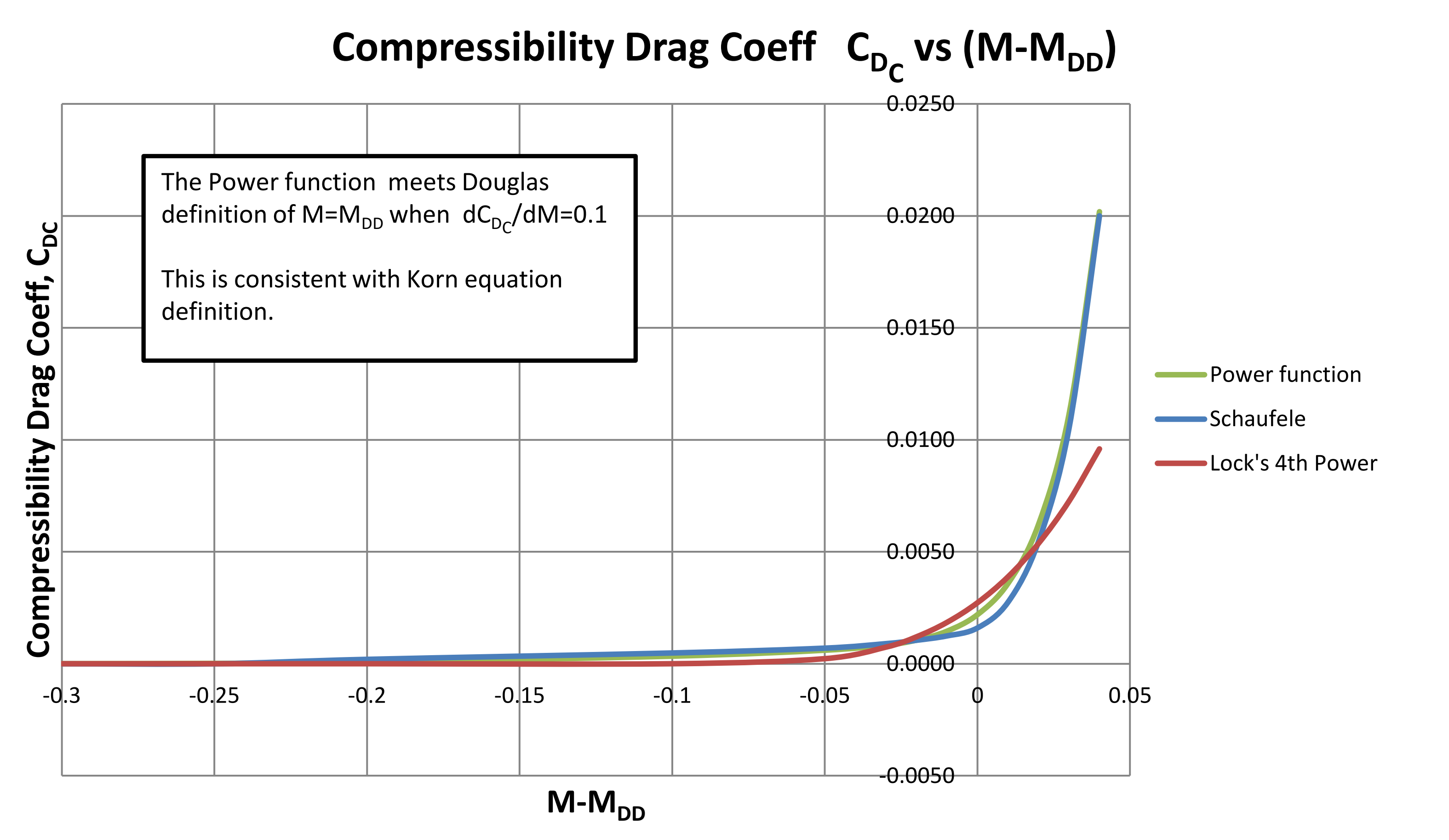
Empirical Estimate of Drag Rise



Alternative Method of M_{DD} Estimation

Empirical Korn Equation applied to airfoil section

$$M_{DD} = \frac{k_a}{\cos\left(\Lambda_{\frac{c}{2}}\right)} - \frac{\frac{t}{c}}{\cos^2\left(\Lambda_{\frac{c}{2}}\right)} - \frac{C_l}{10\cos^3\left(\Lambda_{\frac{c}{2}}\right)}$$

Douglas definition

where

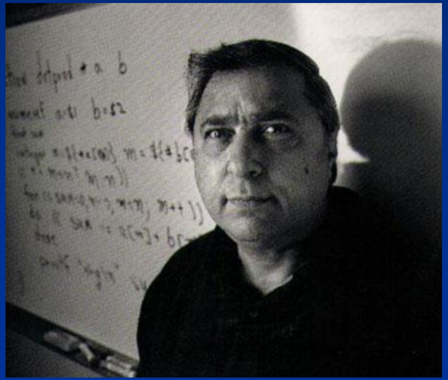
k_a = technology factor

$(= 0.87$ for NACA 6-series$)$

$(= 0.95$ for supercritical airfoil$)$

For wing, divide into sections and average results

Equation developed by Dave Korn at NYU



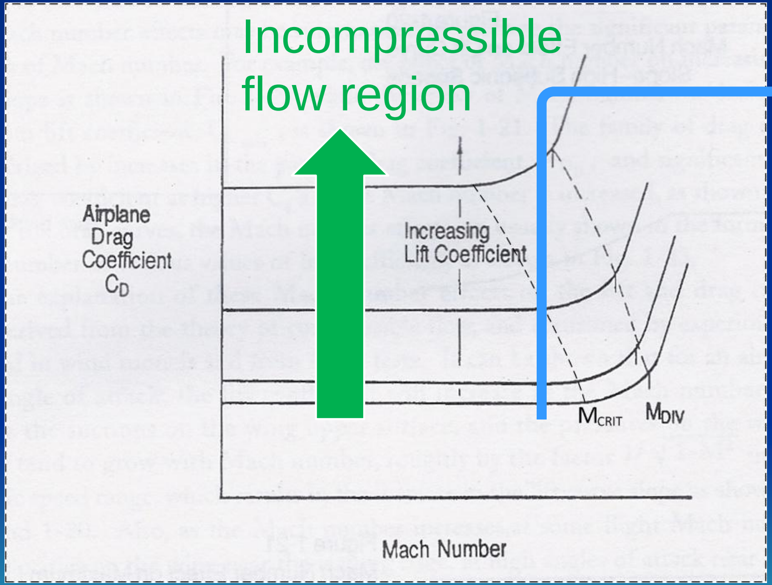
For this approximation, use average values for whole wing

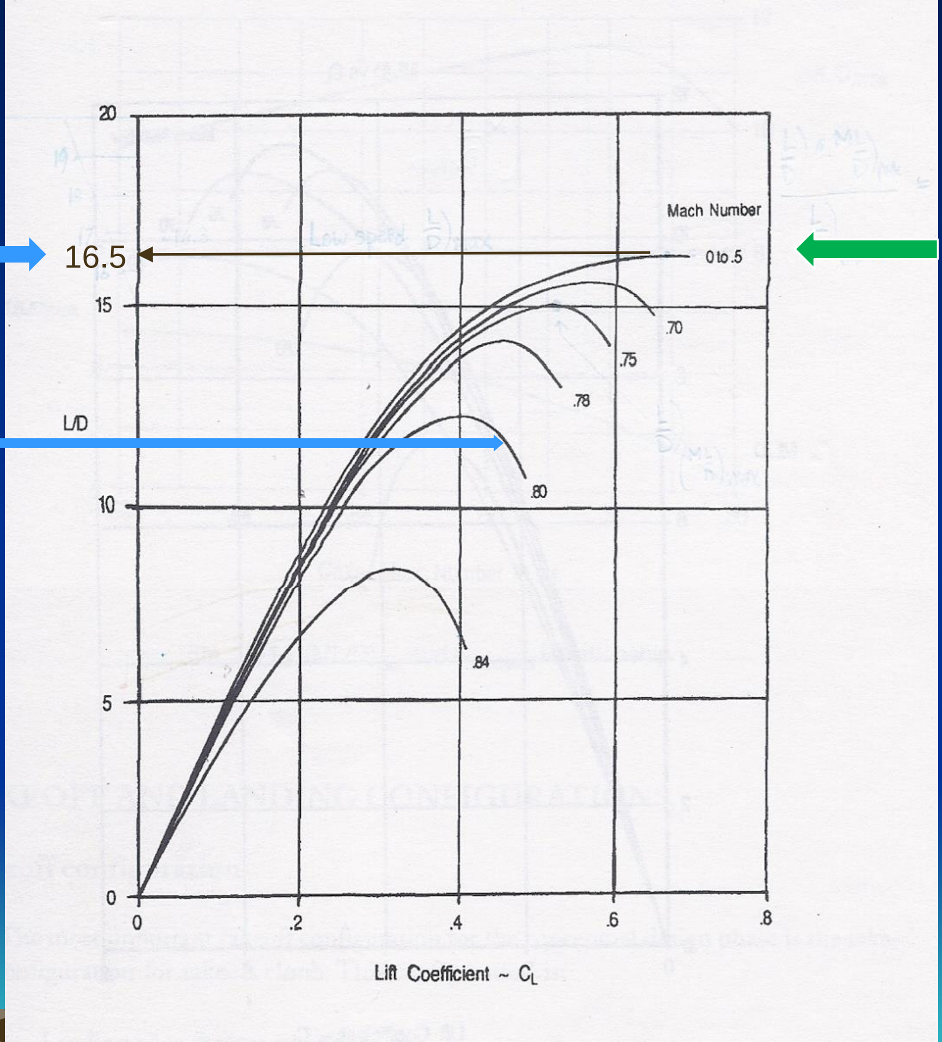
DC-9 Lift/Drag Ratio vs. C_L

- Max low speed L/D =16.5

Incompressible flow region

Take vertical slice through drag map





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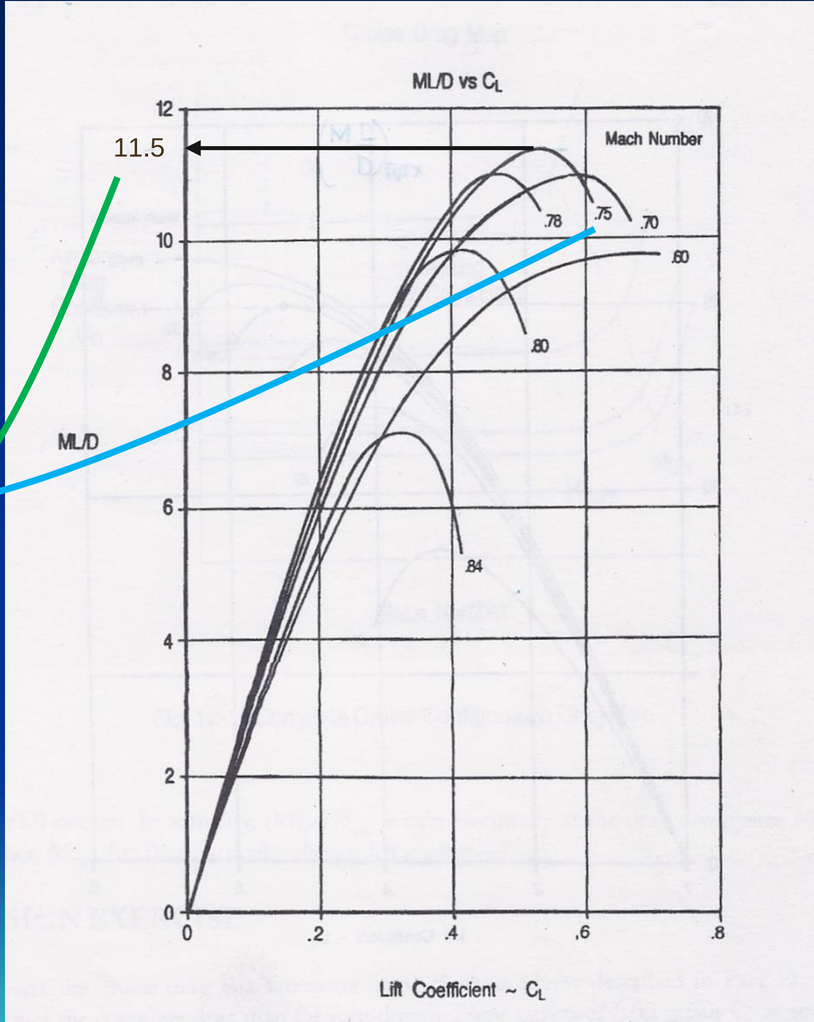
DC-9 ML/D vs C_L

- DC-9 airfoil is not supercritical
- $(M\ L/D)_{max}$ occurs at about M = 0.75
- $(M\ L/D)_{max}$ = 11.5

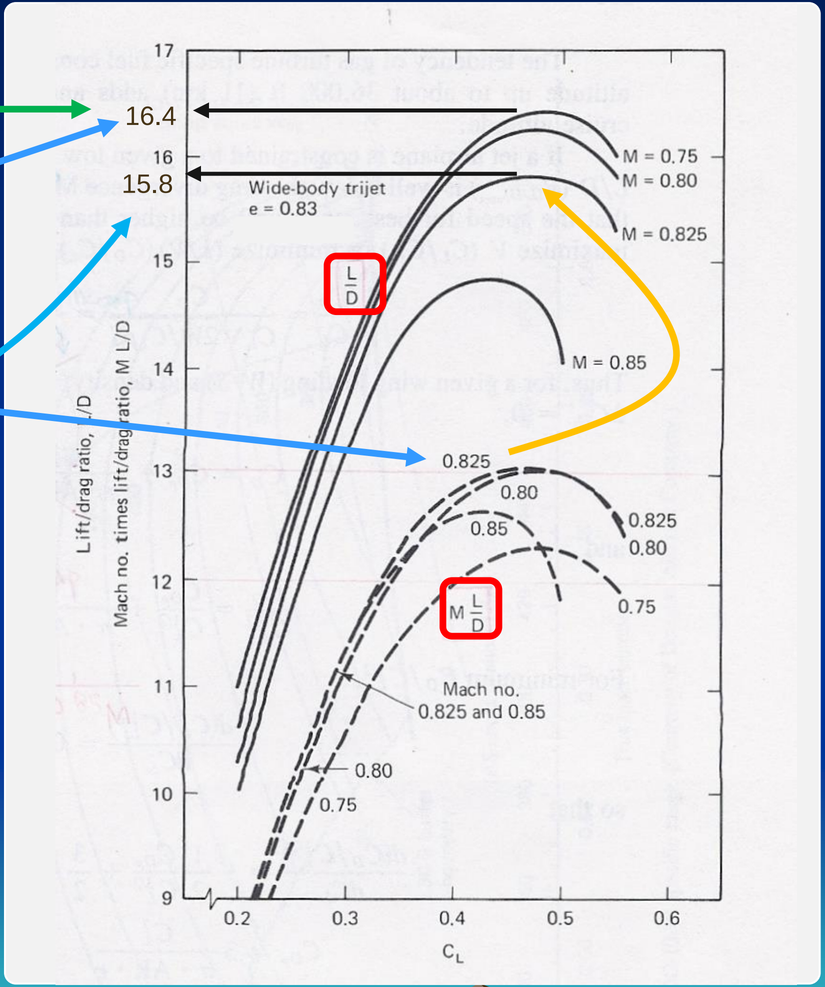


Source: Schaufele

DC-10 L/D and (M L/D)

For incompressible flow

- $((L/D)_{max})_{M=0.75} = 16.4$
- $(M\,L/D)_{max}$ occurs at M=0.825
- $(L/D)_{M=0.825} = 15.8$
- $(L/D)_{M=0.825}/(L/D)_{max\ incomp}$ = 15.8/16.4 = 0.96

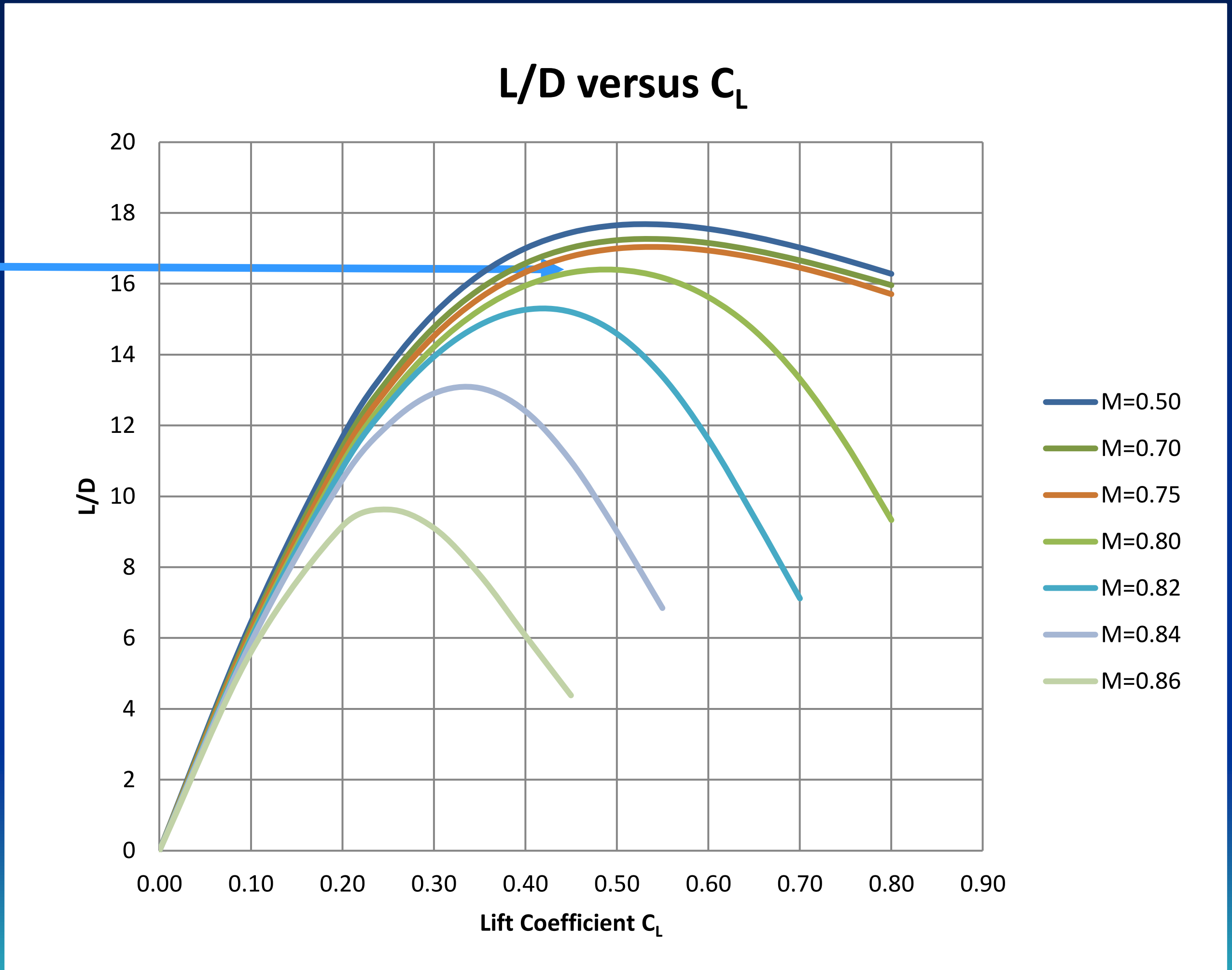


Source: Shevell

Raymer claims $(L/D)/(L/D)_{max\ incomp}$ = 0.86 but that's not always the case

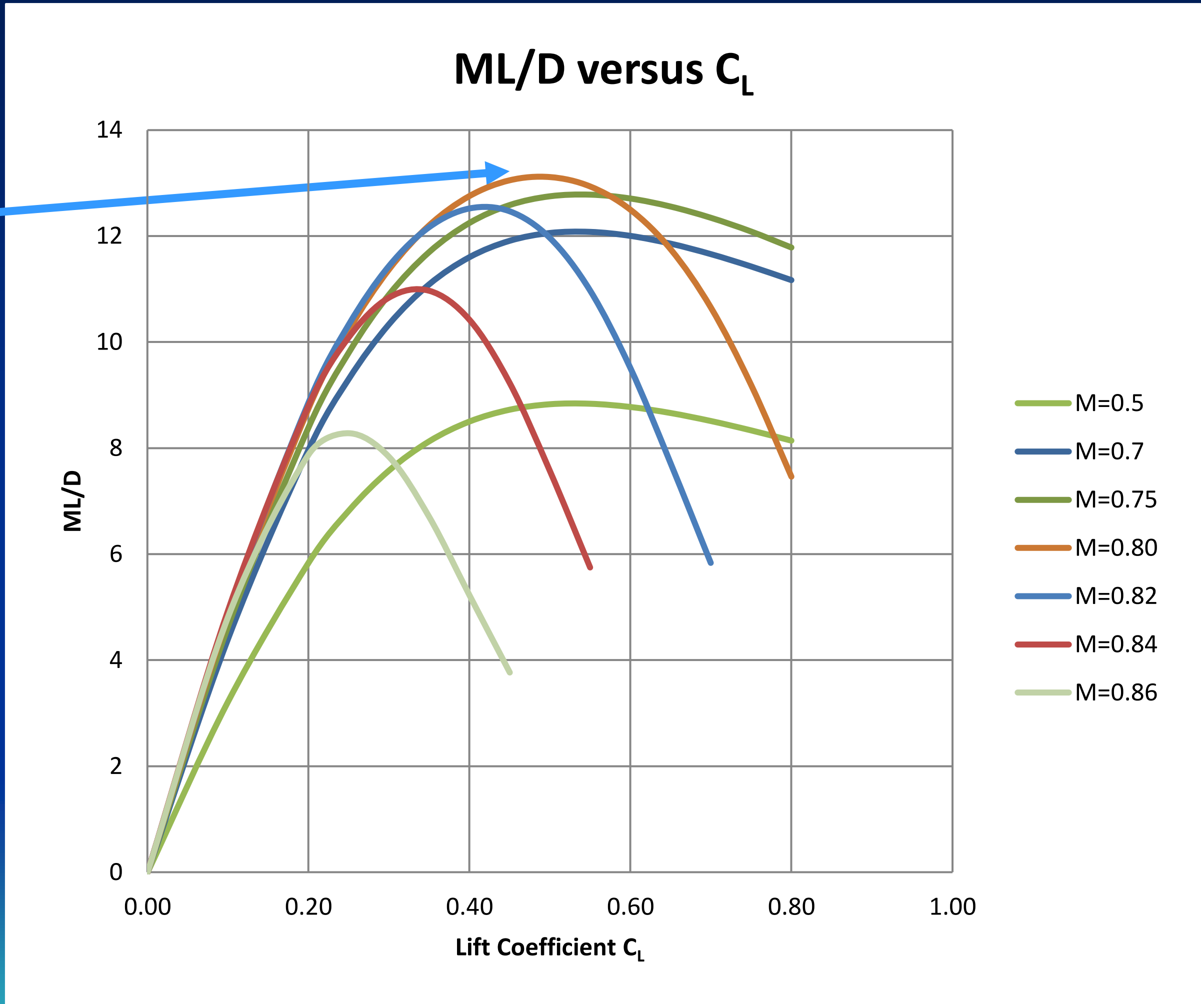
Spreadsheet Prediction for DC-10

$((L/D)_{max})_{M=0.8} = 16.4$



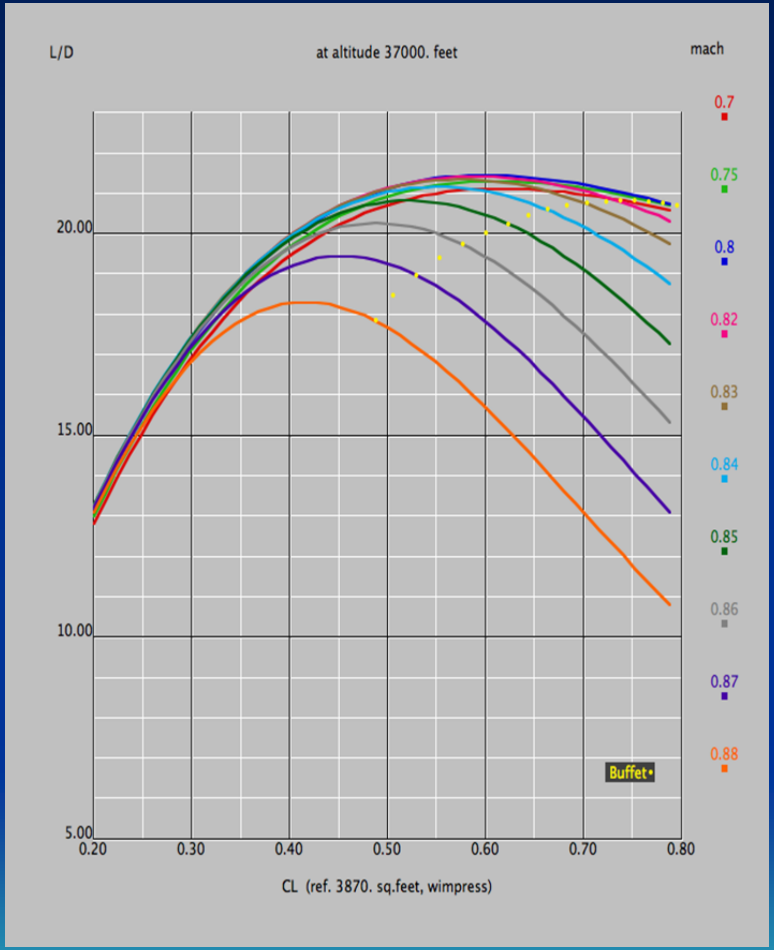
Spreadsheet Prediction for DC-10

- $M_{(ML/D)max} = 0.80$
- $(M\ L/D)_{max} = 13$



Piano Prediction for 787

Piano is European industrial-grade sizing and performance program



Wave Drag due to Lift C_{D_W}

Subsonic/Transonic

Supersonic

Wave Drag due to Lift C_{D_W}

- Subsonic/Transonic
- Supersonic
  - Graphical
  - Empirical Equation
  - Leading Edge Suction

Supersonic Drag due to Lift

Drag due to lift =
Incompressible drag due to lift
+ Wave drag due to lift

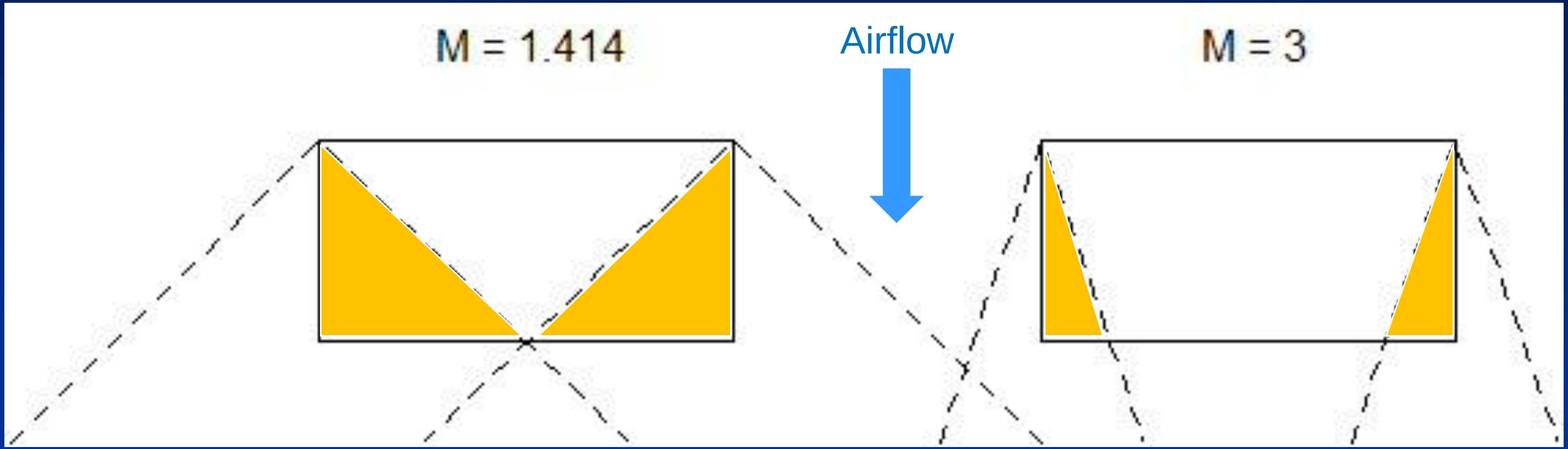
e is Oswald efficiency factor

K includes both subsonic and supersonic drag due to lift and is a function of Mach number

$$C_{D_{lift}} = C_{D_i} + \left(C_{D_w}\right)_{lift}$$
$$= \frac{1}{\pi A e} C_L^2 + \left(C_{D_w}\right)_{lift}$$
$$= K C_L^2$$

where K = Drag due to lift factor

Cones of Influence for AR=2 Wing



- As M increases, area of wing influenced by wingtips decreases and linear theory dominates

Supersonic Estimation of K

Leading edge suction method is more accurate, but required inputs may not be available during conceptual design

Empirical Equation

$$K = \frac{A\left(M^2 - 1\right)\cos\Lambda_{LE}}{\left(4A\sqrt{M^2 - 1}\right) - 2}$$

where

$A = \text{aspect ratio}$

$M = \text{Mach number}$

$\Lambda_{LE} = \text{wing leading edge sweep}$

Raymer Eq. 12.51

Estimation of K for Delta Wing Config.

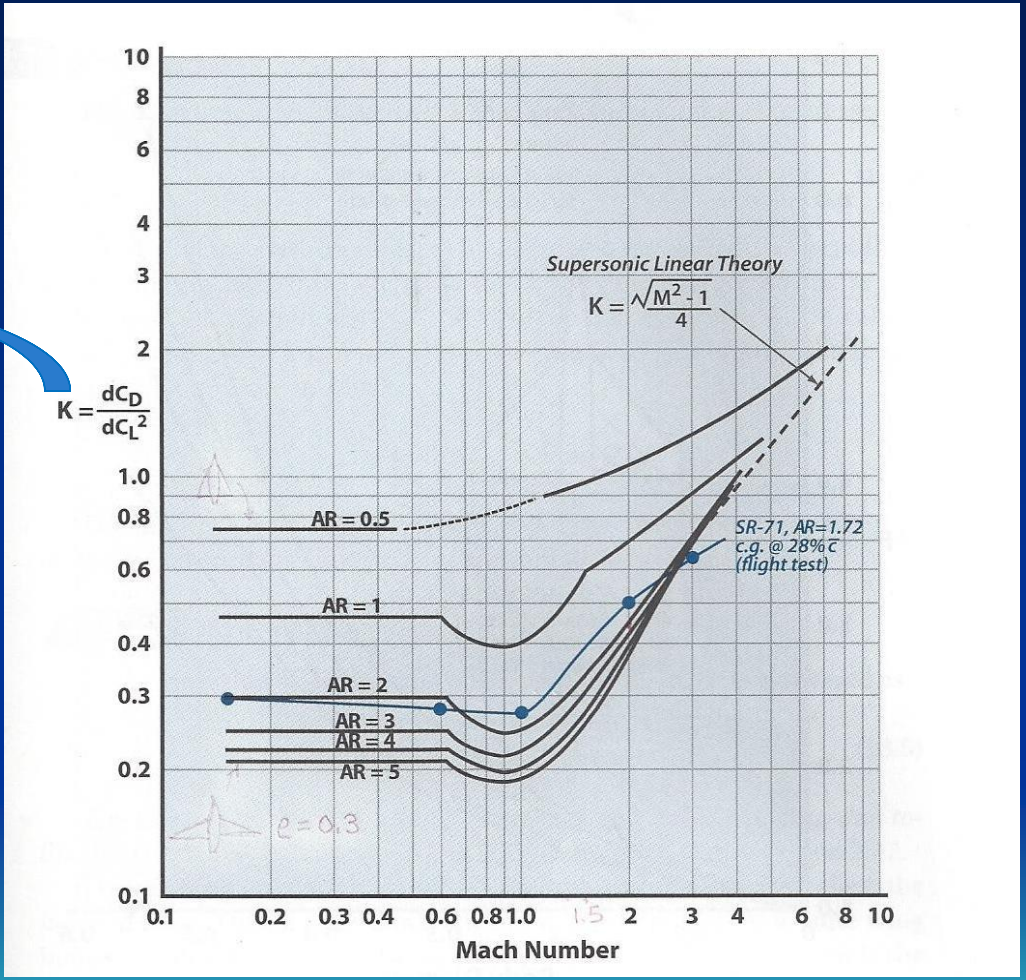
In equation for drag polar

$$C_D = C_{D_0} + KC_L^2$$

In this figure:

fuselage with delta wing

with l.e. radius $= 0.045\%$



Source: Nicolai & Carichner Fig 13.3b

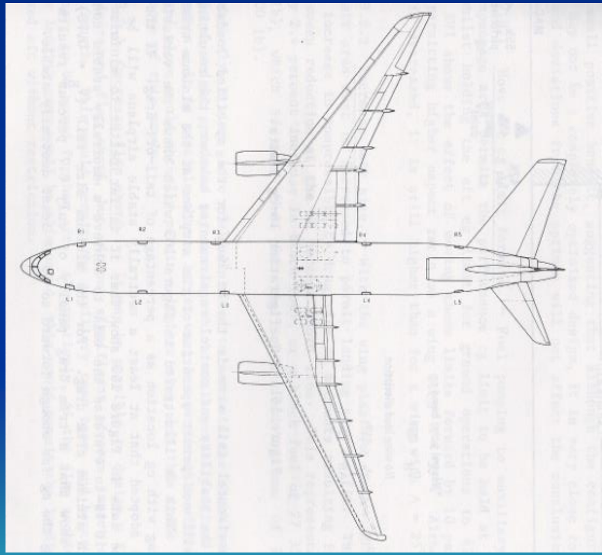
Aerodynamic Analysis

To Summarize - this is what we covered:

- Lift and High Lift Systems
- Zero-Lift Drag C_{D_0}
- Drag due to Lift C_{D_i}
- Wave Drag due to Volume $C_{D_{0_{supersonic}}}$
- Wave Drag due to Lift C_{D_W}

Wing Design Trades Λ = 30°

- For unconstrained design, 30° sweep is slightly better



Source: NASA CR3586

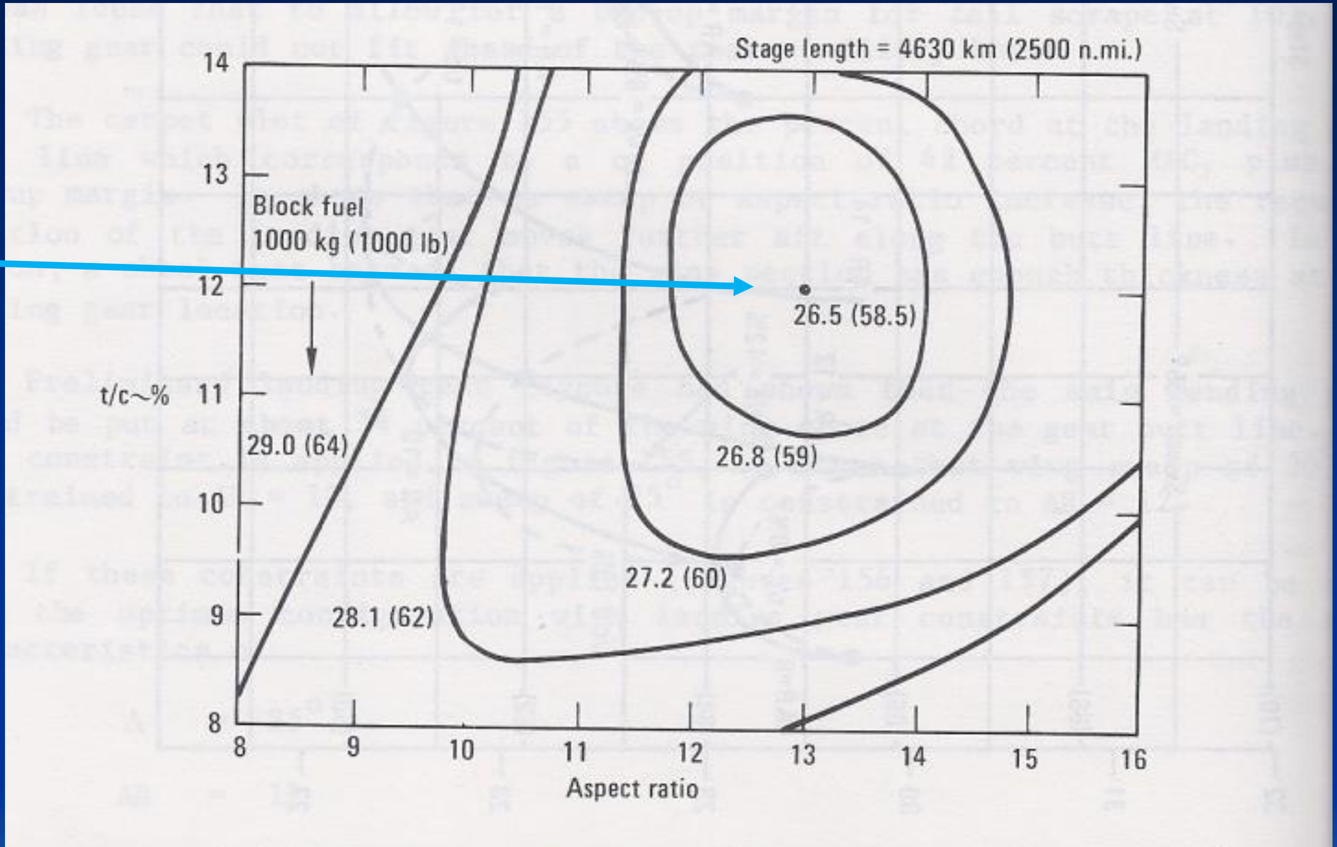


Figure 154. - Configuration P16 - block fuel knothole, Λ = 30°.

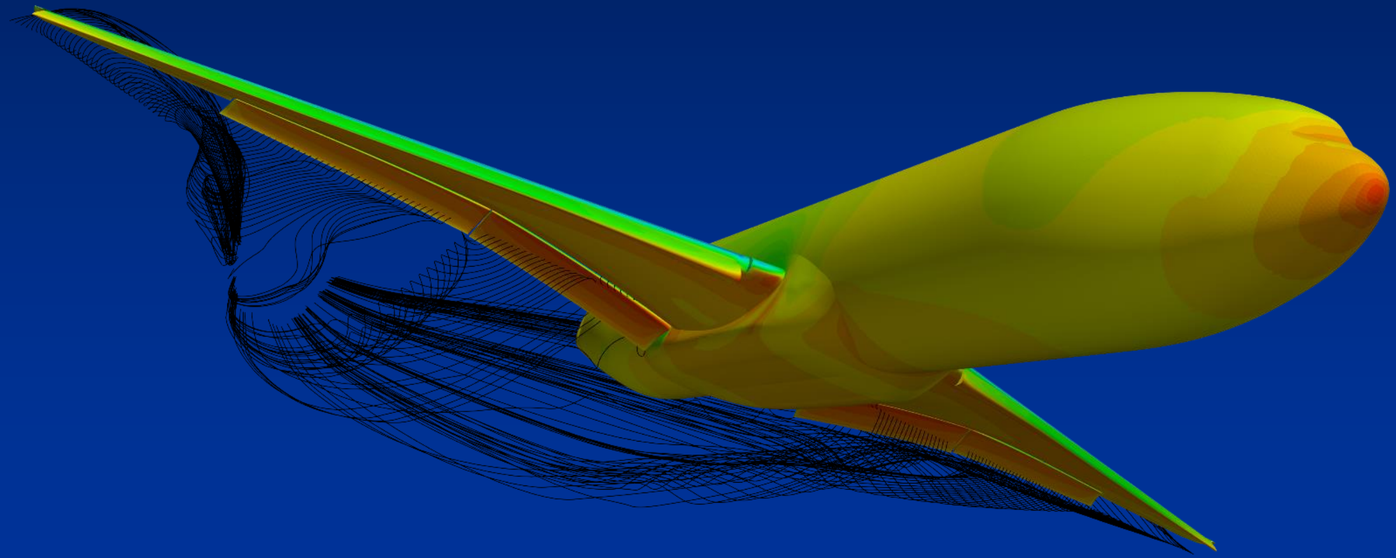
Source: NASA CR3586

SU2 Open Source CFD Analysis

- Solves Multiphysics analysis and optimization tasks
- Unstructured mesh topology
- Use to provide optimal shape design using gradient-based framework
- Goal-oriented adaptive mesh refinement
- See AIAA paper

Thomas D. Economon, Francisco Palacios, Sean R. Copeland, Trent W. Lukaczyk and Juan J. Alonso

SU2: An Open-source Suite for Multiphysics Simulation and Design (AIAA Journal, Vol 54, Number 3, March 2016)

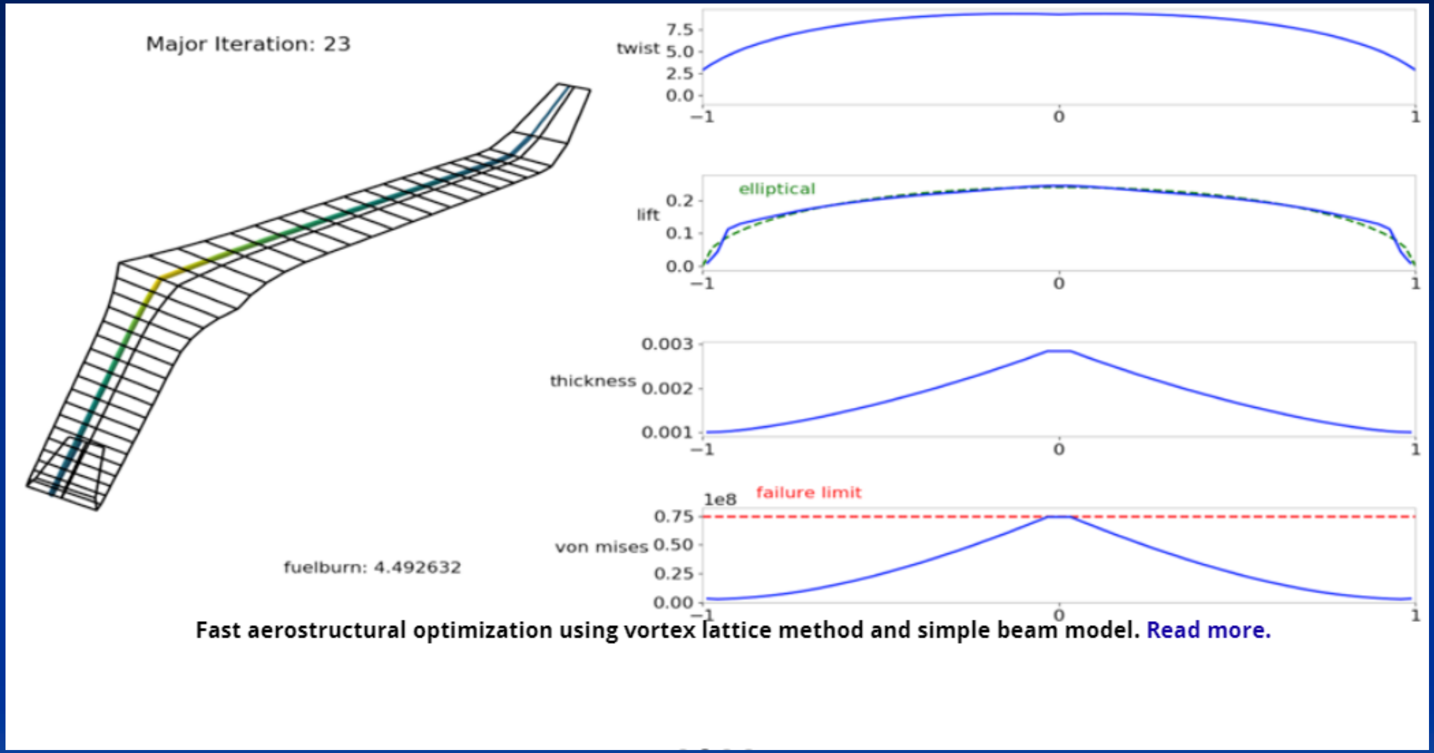


OpenMDAO

- Multidisciplinary Analysis and Optimization
- Developed at NASA-Glenn Research Center
- Written in Python

J. S. Gray, J. T. Hwang, J. R. R. A. Martins, K. T. Moore, and B. A. Naylor, "OpenMDAO:

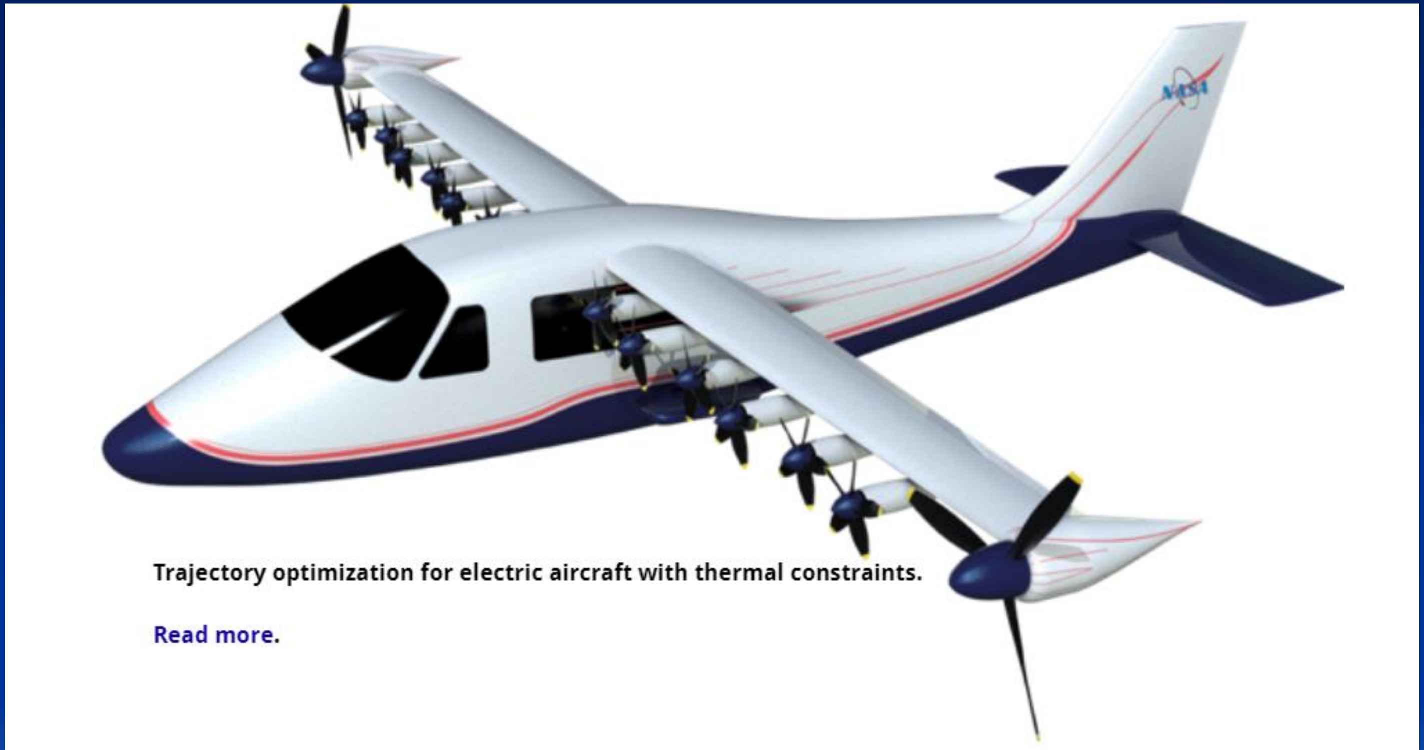
An Open-Source Framework for Multidisciplinary Design, Analysis, and Optimization," Structural and Multidisciplinary Optimization, 2019.



Fast aerostructural optimization using vortex lattice method and simple beam model. **Read more.**

OpenMDAO

- Multidisciplinary Analysis and Optimization
- Developed at NASA-Glenn Research Center
- Written in Python



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Development of a Multi-Phase Mission Planning Tool for NASA X-57 Maxwell
http://openmdao.org/pubs/x57_mpt_2018.pdf

OpenMDAO

- Incorporates older version of SU2

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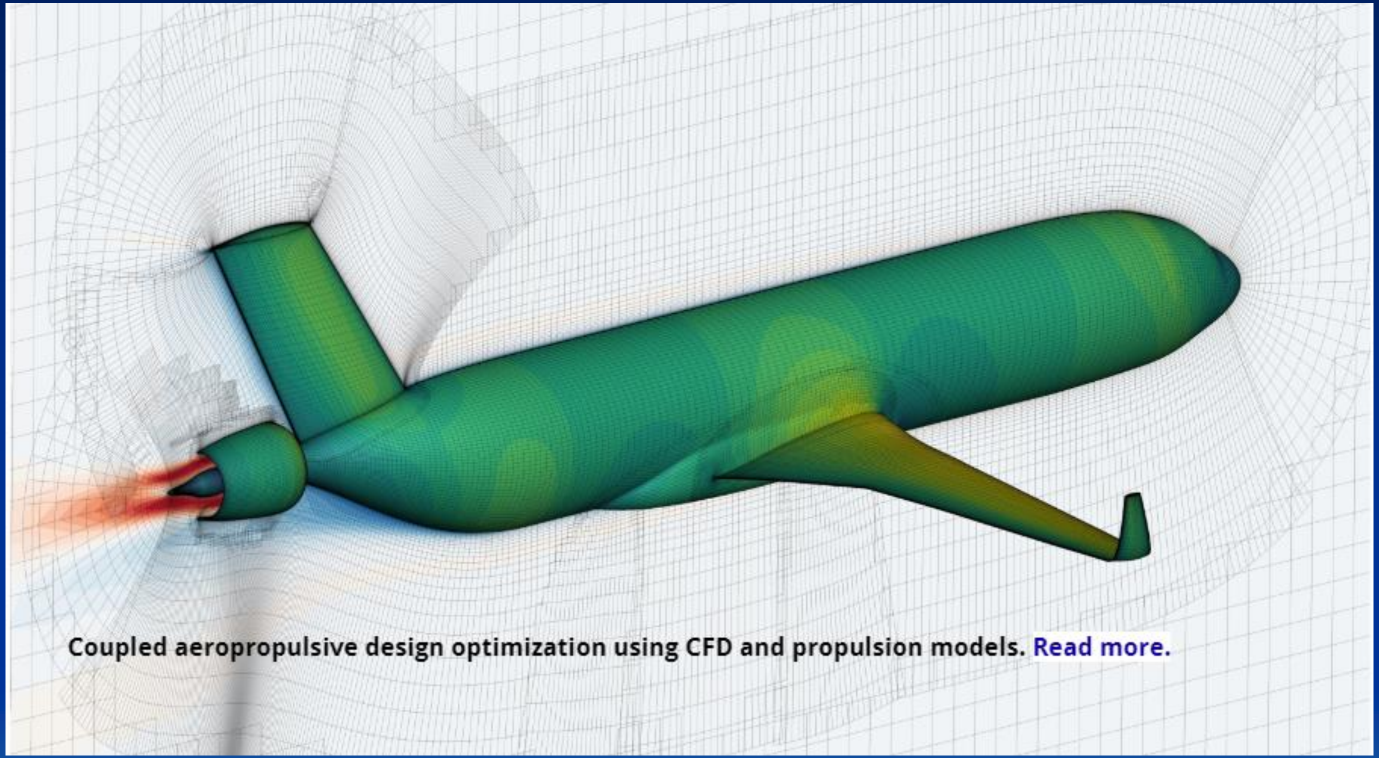
Aero propulsive Design Optimization of a Turboelectric Boundary Layer Ingestion Propulsion System



Aerodynamics

What did we cover:

- Lift and High Lift Systems
- Zero-Lift Drag C_{D_0}
- Drag due to Lift C_{D_i}
- Wave Drag due to Volume $C_{D_{0_{supersonic}}}$
- Wave Drag due to Lift C_{D_W}