17.8.4 Balanced Field Length

Raymer Eq. (17.113) does not provide any insight as how balanced field length is determined. The following method and example, taken from Roger Schaufele's "The Elements of Aircraft Preliminary Design" (Ref. 17.8.4.1), describe the process for estimating balanced field length, and hence FAR field length, and is shown in a tabular form here, as described in Schaufele's book. Values have mostly been copied verbatim, although there appear to be numerous minor arithmetic errors, such as the factor to correct knots to ft/sec, which has been corrected. They are a slightly simplified version of the method described in FAR 25.113. This procedure can be more simply set up in a spreadsheet, as illustrated in the spreadsheet "Balanced Field Length DC-9" at https://www.adac.aero/design-data, or as a conventional computer program.

FAR required field length for multi-engine commercial operations is the greater of

- Balanced field length.
- (All engines operational (AEO) takeoff distance to 35 ft.) x 1.15

There are three possible events for takeoff

1. All engines operational takeoff
2. One engine inoperative (OEI) takeoff
3. One engine inoperative accelerate – stop

The following distances and speeds are defined as

S_{Ground}	Ground distance to liftoff
$S_{Ground1}$	Ground distance to V_1
$S_{Ground2}$	Ground distance from V_1 to V=0 (rejected takeoff)
S_a	Air distance to 35 ft screen height
V_{EF}	Engine failure speed
V_{LOF}	Airplane liftoff speed
V_R	Speed at which the pilot rotates the airplane for takeoff
$V_{S_{TO}}$	Stall speed with flaps in the takeoff position
V_1	"Decision" speed (actually "commit to fly" speed)
V_2 (= V_{35})	Speed at screen height of 35 ft with critical engine inoperative, must be greater than or equal to 1.2 $V_{S_{TO}}$

Other abbreviations are

AEO	All engines operating
D	aircraft drag (lb)
F_r	Total rolling resistance of landing gear
h	Height above ground (ft)
OEI	One engine inoperative
RTO	Rejected takeoff
TOFL	Takeoff field length (ft)
W	Aircraft weight (lb)
γ	Climb angle (radians)
μ_R	Rolling coefficient of friction

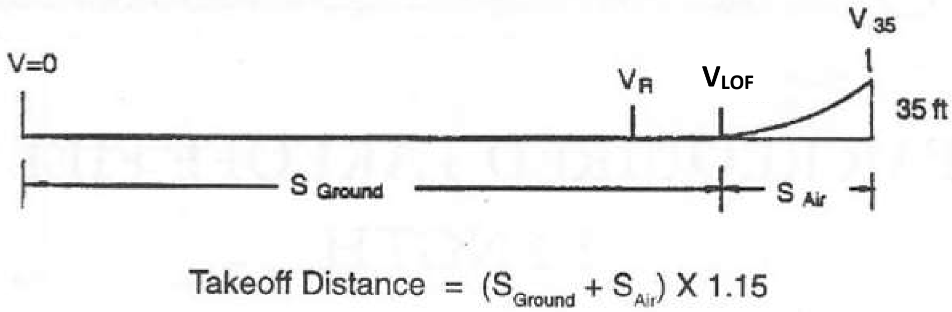

Fig. 17.8.4.1 All engines operating takeoff

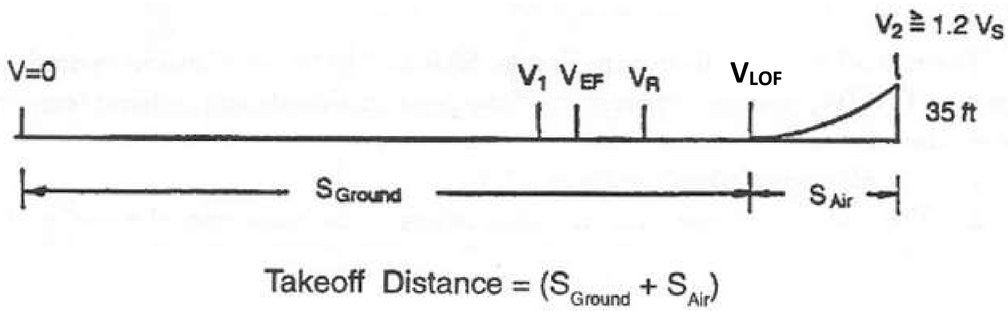

Fig. 17.8.4.2 One engine inoperative takeoff

If engine failure recognition occurs after V_1, then the pilot flying the airplane (PF) must continue the takeoff, because there may be insufficient runway length remaining to stop.

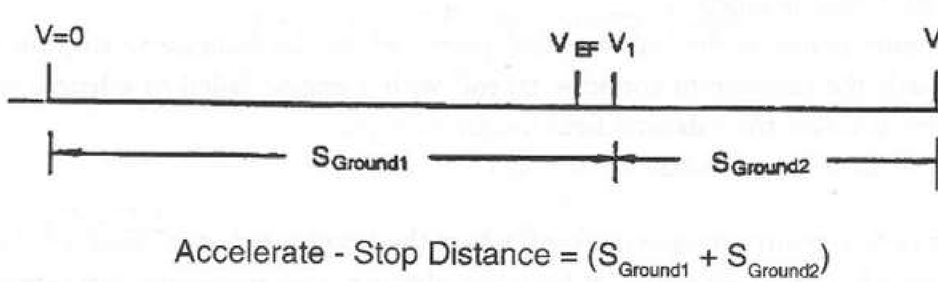

Fig. 17.8.4.3 One engine inoperative accelerate-stop

If engine failure recognition occurs before V_1 then the PF must abort the takeoff, because the aircraft may not accelerate sufficiently to reach 35 ft at the end of the runway.

- Balanced field length requires that TOFL for events 2 and 3 be equal
- FAA Required field length is the greater of 1 and 2 (=3)

Balanced field length will be considered first.

The FAA definition of V_1, as described in Ref. 17.8.4.2 is as follows:
"V_1 means the maximum speed in the takeoff at which the pilot must take the first action (e.g., apply brakes, reduce thrust, deploy speed brakes) to stop the airplane within the accelerate-stop distance. V_1 also means the minimum speed in the takeoff, following a failure of the critical engine at V_{EF}, at which the pilot can continue the takeoff and achieve the required height above the takeoff surface within the takeoff distance".

V_1 is often called the "decision speed", but that is somewhat of a misnomer. There is no decision. The PF either stops the airplane if engine failure is recognized before V_1, or continues takeoff if engine failure is recognized after V_1. During takeoff roll, if there is no annunciator on the flight deck, the pilot not flying (PNF) will call out "V-1","Rotate", and "V-2" when the appropriate speeds are reached.

The FAA definition of takeoff distance and takeoff run can be found in Ref. 18.8.4.3. §25.113. The definition of accelerate-stop distance is a slight simplification of the FAR §25.109 requirement, which adds a distance equivalent to 2 seconds at V_1, to account for the difference between engine failure and action.

"Most critical point" in a given takeoff, where the "accelerate-stop" distance is equal to the "accelerate-continue" distance, is found by plotting "accelerate-stop" distance and "accelerate-continue" with one engine failed, versus the engine failure recognition speed.

For propeller-driven transports, the FAR balanced field length concept is the same, but the speed for the one-engine inoperative distance is 1.15 V_S at 35 ft for four or more engines, or 1.2 Vs for two or three engines.

For multi-engine military aircraft, the required field length is specified in MIL-C-5011B, and the balanced field length is called "critical field length", and the accelerate-continue distance is to lift-off, not to 35 ft above the runway.

For single- and multi-engine normal, utility and acrobatic aircraft under 12,500 lb maximum gross weight, takeoff requirements can be found in FAR Part 23 (Ref. 17.8.4.4).

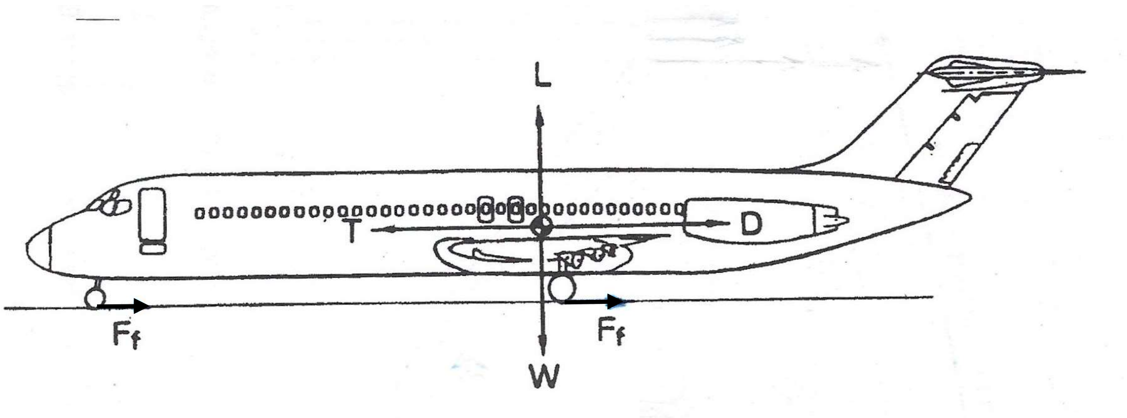

Fig 17.8.4.4 Force diagram during ground run

From Fig. 17.8.4.4, acceleration during ground roll can be expressed as

$$a = \frac{g}{W}\left(T - D - F_r\right) = \frac{g}{W}\left(T - D - \mu_R\left(W - L\right)\right) \qquad \text{Eq. 17.8.4.1}$$

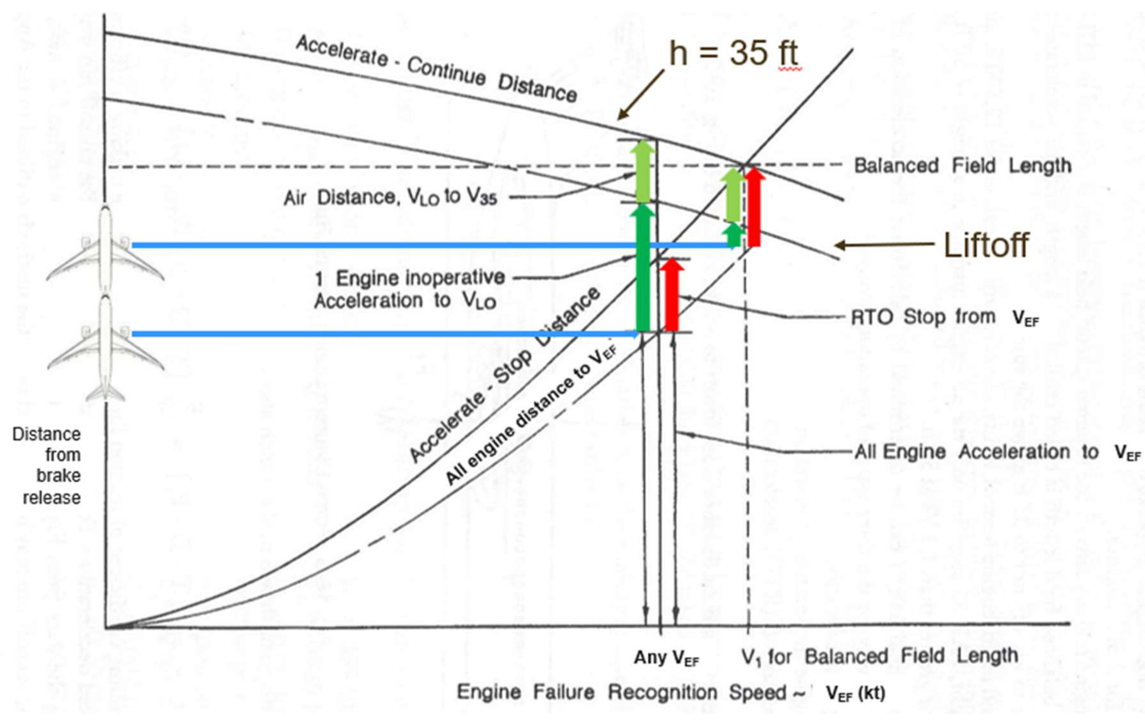

Fig. 17.8.4.5 Balanced field length concept

From Fig. 17.8.4.5 it can be seen that engine failure at V_1 is at the worst possible time. Before V_1, the airplane can be stopped in a shorter distance than at V_1. After V_1, the airplane can continue to 35 ft at a shorter distance than at V_1.

EXAMPLE PROBLEM

Construct a balanced field length chart for a short-range jet transport at sea level, standard day conditions, TOGW = 100,000 lb, flaps 15 deg for takeoff, JT8D-7 engines

From data given below, calculate the acceleration, a, on the runway at several speeds, from zero to beyond V_2 for three types of situation involved

- All-engine acceleration
- One-engine-inoperative (OEI) acceleration
- Rejected takeoff (RTO) deceleration

Assumptions for calculation of acceleration for the three conditions are in Table 17.8.4.1. Because of the proximity of the wing to the ground, induced drag can be neglected during ground roll.

Condition	Data value
Airplane lift coeff. in rolling attitude	C_L= 0.355 (given)
Airplane drag coeff. in rolling attitude	C_D = 0.0585 (given)
Rolling coefficient of friction	μ_R = 0.02 (given)
Thrust/engine	From engine data
Reference wing area	S = 1000 ft^2 (given)
Airplane lift in RTO configuration	C_L = 0 (given)
Airplane drag in RTO configuration	C_D = 0.1082 (see Table 17.8.4.2)
Braking coefficient of friction	μ_B = 0.02 (given)
At TOGW = 100,000 lb, flaps = 15°	Vs = 110 kt (given)

17.8.4.1 Assumptions for balanced field length estimation

Component	Drag coefficient
Aircraft (clean)	0.0200
Flaps	0.0070
Spoilers	0.0500
Landing gear	0.0250
Slats	0.0062
Total	0.1082

Table 17.8.4.2 Assumed RTO drag buildup for DC-9

For aircraft with spoilers, C_L at RTO configuration may, in practice, be negative. A conservative value of zero is used here. The following three tables must be compiled.

V (kt)	Source	0	40	80	120	140
Mach	0.0015 V	0	0.60	0.120	0.180	0.211
T/eng (lb)	Engine table	13,500	13,200	12,700	12,300	12,100
T_{total} (lb)	2 T/eng	27,000	26,400	25,400	24,600	24,200
C_D	Table 17.8.4.1	0.0585	0.0585	0.0585	0.0585	0.0585
q (lb/ft^2)	1481 M^2	0	5.3	21.3	48.5	65.9
D (lb)	C_D q S	0	310	1250	2840	3860
C_L	Table 17.8.4.1	0.355	0.355	0.355	0.355	0.355
L (lb)	C_L q S	0	1892	7561	17,220	23,400
W (lb)	W	100,000	100,000	100,000	100,000	100,000
W-L (lb)	W-L	100,000	98,110	92,440	82,800	76,600
F_f (lb)	(W-L) μ_R	2,000	1,962	1,849	1,656	1,532
ΣF_{RW} (lb)	T_{total} - F_f	25,000	24,130	22,300	20,100	218,720
a (ft/sec)	Eq.17.8.4.1	8.04	7.79	7.20	6.49	6.05
1/(2a) (ft/sec^2)		0.0622	0.0643	0.0695	0.0771	0.0826
V^2 (ft^2/sec^2)	(1.6878 V(kt))^2	0 (a)	4,558 (b)	18,231 (c)	41,020 (d)	55,834 (e)

For letters in parentheses, see Fig. 17.8.4.6

Table 17.8.4.3 Calculation of acceleration – All engines

V (kt)	Source	0	40	80	120	140
ΣF_{RW} (lb)	T_{total} - F_f	25,000	24,130	22,300	20,000	18,720
T/eng (lb)	Engine table	13,500	13,200	12,700	12,300	12,100
ΣF_{rw} (lb)	ΣF_{RW} – T/eng	11,500	10,930	9,600	7,800	6,620
a (ft/sec)	Eq.17.8.4.1	3.70	3.52	3.06	2.51	2.13
1/(2a) (ft/sec^2)		0.135	0.142	0.163	0.199	0.235
V^2 (ft^2/sec^2)	(1.6878 V(kt))^2	0 (f)	4,558 (g)	18,231 (h)	41,020 (i)	55,834 (j)

For letters in parentheses, see Fig. 17.8.4.6

Table 17.8.4.4 Calculation of acceleration – One engine inoperative

V (kt)	Source	0	40	80	120	140
q (lb/ft^2)	1481 M^2	0	5.3	21.3	48.5	65.9
C_D	Table 17.8.4.1	0.1082	0.1082	0.1082	0.1082	0.1082
D (lb)	C_D q S	0	580	2305	5250	7130
W-L (lb)	W-L	100,000	100,000	100,000	100,000	100,000
F_f (lb)	(W-L) μ_R	30,000	30,000	30,000	30,000	30,000
ΣF_{RW} (lb)	T_{total} - F_f	25,000	24,130	22,300	20,100	218,720
a (ft/sec)	Eq.17.8.4.1	-9.65	-9.89	-10.4	-11.35	-11.95
1/(2a) (ft/sec^2)		-0.0518	-0.0510	-0.0431	-0.440	-0.0418
V^2 (ft^2/sec^2)	$(1.6878\ V(kt))^2$	0 (k)	4,480 (l)	17,950 (m)	40,800 (n)	55,450 (o)

For letters in parentheses, see Fig. 17.8.4.6

Table 17.8.4.5 Calculation of deceleration – Rejected Takeoff

Determination of Accelerate-Stop Distances

From the equation of motion

$$V^2 = U^2 + 2as \qquad \text{Eq. 17.8.4.2}$$

$$\text{Distance} \quad s = \frac{V^2 - U^2}{2a} \qquad \text{Eq. 17.8.4.3}$$

The procedure described here may appear to be somewhat convoluted, and it is. In Schaufele's book, the values of 1/(2a) are plotted as a function of V^2. The area under the curve is the distance travelled. But since the curves are close to a straight line, the values are approximated by taking the product of V^2 and 1/(2a). For example, to calculate a stop from 40 kt, from Table 17.8.4.5

s = 0.051 x 4,480 = 230 ft (numbers rounded to nearest 10 ft)

This is somewhat of a simplification. Each area in Fig. 17.8.4.6 is treated as a parallelogram, whereas it is actually a trapezoid. Each distance is therefore slightly conservative, i.e., the distance is overstated. For a given velocity, V_n, a more accurate value for the area would be

$$\text{Distance} \quad s = \left(\left(V_n\right)^2 - \left(V_{n-1}\right)^2 \right) \times \frac{1}{2} \left(\frac{1}{2a_{n-1}} + \frac{1}{2a_n} \right) \qquad \text{Eq.17.8.4.4}$$

where V_{n-1} = U in Eq. 17.8.4.3. However, this was not done in Schaufele's example.

V (kt)		0	40	80	120	140
Accel to V_{EF} (ft)		0	290 (B)	1150	2860	4040
Stop from V_{EF}(ft)		0	230 (A)	870	1950	2580
Accel-stop (ft)		0	520	2020	4310	6620

For letters in parentheses, see Fig. 17.8.4.6

Table 17.8.4.6 Determination of accelerate-stop distance

Determination of Accelerate-Go Distances to V_2

The accelerate-go distance to V_{LO} on the runway can be found from Eq. 17.8.4.3. The air part of the accelerate-go distance is found from empirical correlation based on flight test data involving the time required to go from liftoff to to 35 ft, and the speed increase from liftoff to 35 ft. These parameters are shown in Fig. 16.8.4.6 as S_a/V_{LO} and V_{35}/V_{LO} as a function of out of ground effect (OGE) climb gradient in the takeoff configuration.

The minimum OGE climb gradient for twin-engine transport is 2.4%, as indicated in Raymer Table F.4. This is usually the performance constraint that determines the T/W of the aircraft, so this is a reasonable value to use.

For the DC-9, with takeoff flap setting

- From lower chart of Fig. 16.8.4.6, for gradient of 2.4%, V_{35}/V_{LOF} = 1.012
- V_{Sto} = 110 kt (Table 17.8.4.1)
- V_{35} (= V_2) = 132 kt, so
- V_{LOF} = 132/1.012 = 130 kt = 130 x 1.6878 ft/sec = 220 ft/sec
- From upper chart of Fig. 16.8.4.6, for gradient of 2.4%, S_a/V_{LOF} = 6.2 sec
- S_a = 6.2 x 220 ft/sec = 1360 ft
- This value is used in Table 17.4.8.7

V (kt)		0	40	80	120
Accel to V_{EF} (ft)		0	290 (B)	1,150	2,860
Continue to V_{LO} (ft)		8,190	7,580 (C)	5,700	1,370
Air distance (ft)		1,360	1,360	1,360	1,360
Accel-go (ft)		9,550	9,230	8,120	5,590

For letters in parentheses, see Fig. 17.8.4.6

Table 17.8.4.7 Determination of accelerate-go distance

Determination of Balanced Field Length

The results in Tables 17.8.4.6 and 17.8.4.7 can be plotted as shown in Fig. 17.8.4.7. The curves represent the accelerate-go and accelerate-stop curves in Fig. 17.8.4.5. Their intersection is at the balanced field length

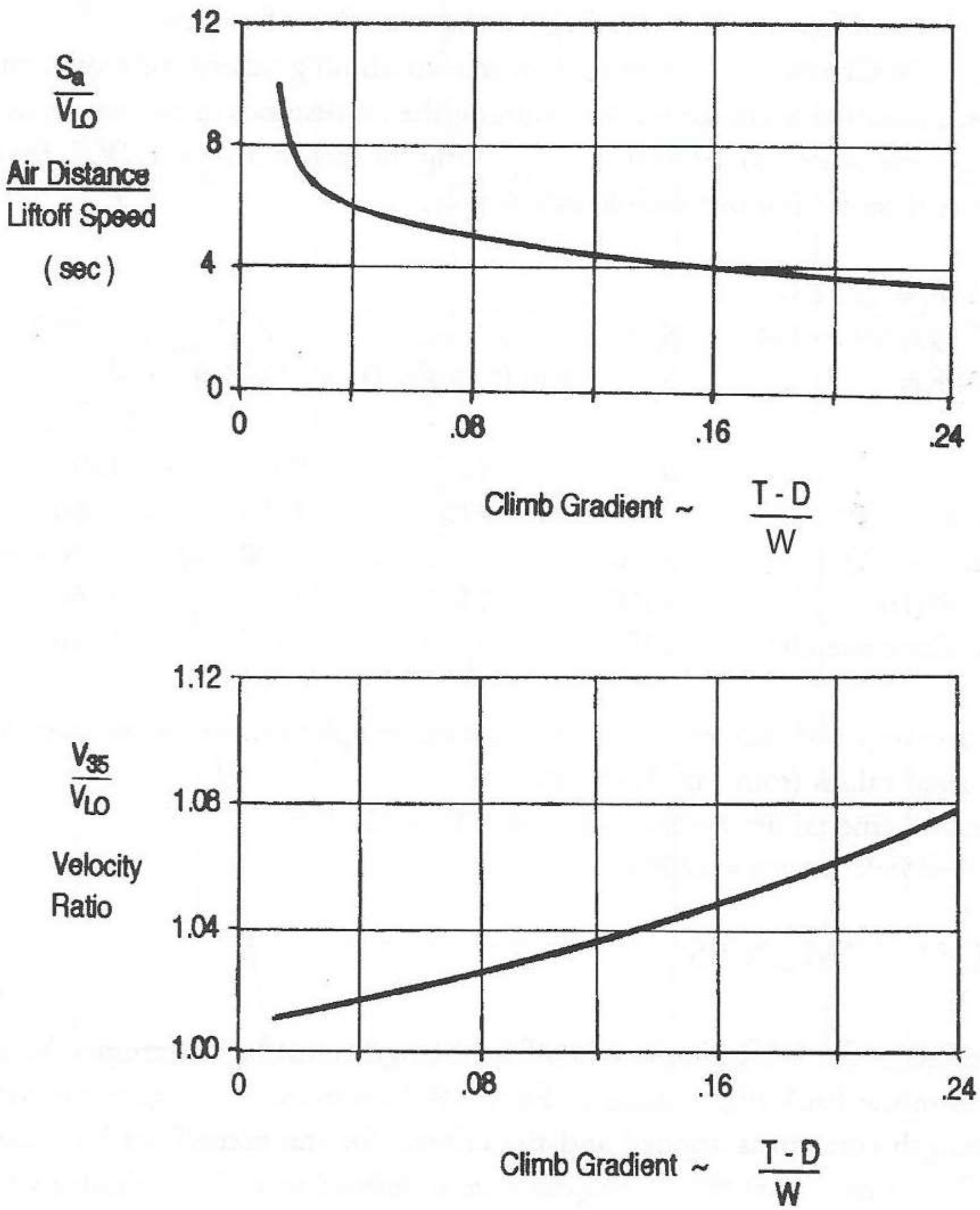

Figure 17.8.4.6 FAR 25 Air Distance Parameters

For aircraft certified under FAR Part 25, required climb gradients at the screen height of 35 ft with one engine inoperative are:

- 2 engines: 2.4%
- 3 engines: 2.7%
- 4 engines: 3.0%

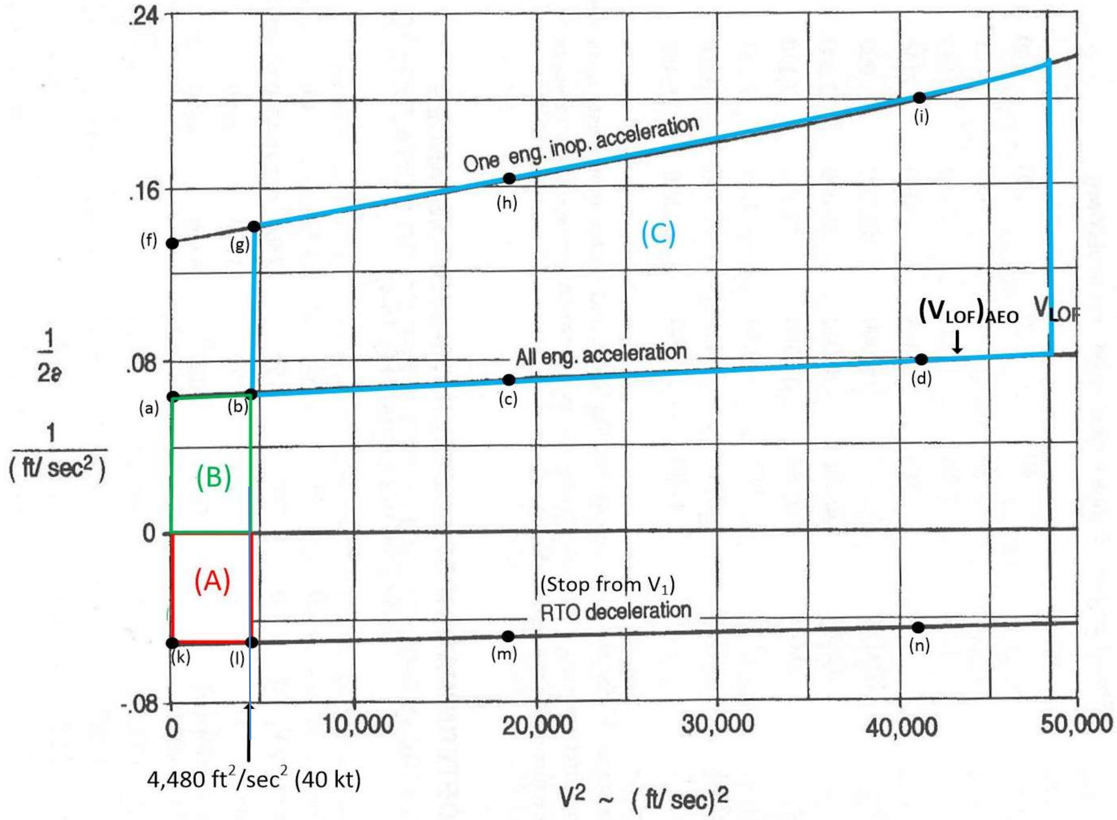

Fig. 17.8.4.6 Takeoff Acceleration-Deceleration Chart

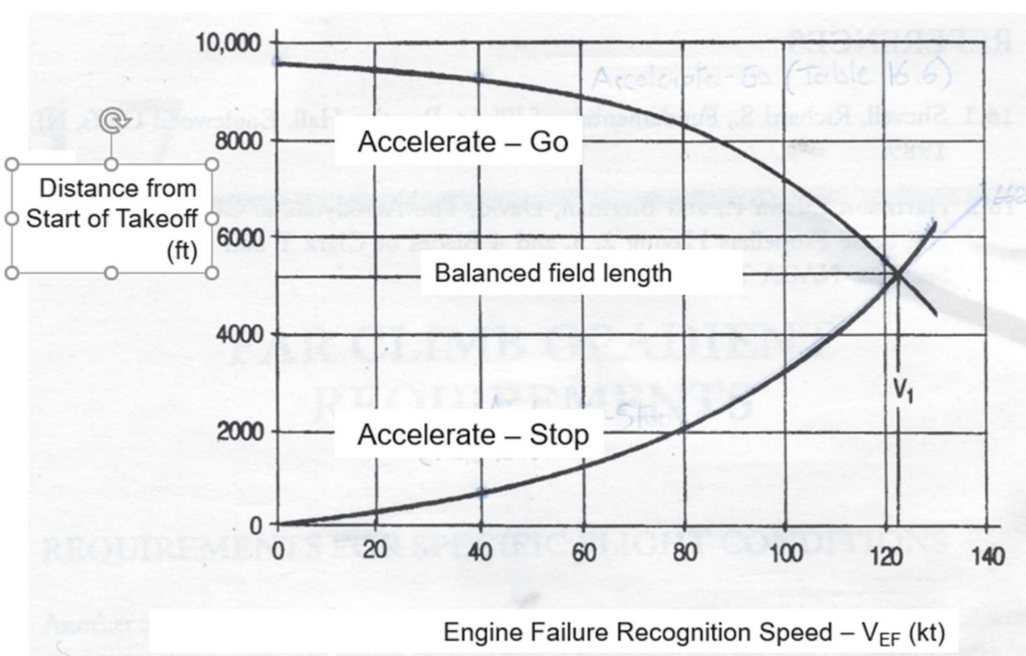

Fig. 17.8.4.7 Takeoff Accelerate-Go and Accelerate-Stop

Determination of All Engines Operating Takeoff to 35 ft

- Assume V_2 (= V_{35}) as for OEI
- Calculate thrust and drag at 35 ft and thus calculate climb angle, $\gamma = (T-D)/W$. Drag must be calculated out of ground effect (FAR 25.121(b)).
- Use Fig. 17.4.8.6 to determine new V_{LOF} (lower half of figure) and new S_A (upper half of figure)
- From Fig. 17.8.4.6 (or spreadsheet), calculate S_G to new V_{LOF}
- Factored takeoff distance, all engines operating = (S_G+S_A) x 1.15

As stated earlier, the FAR required field length for multi-engine commercial operations is the greater of

- Balanced field length.
- (All engines operational (AEO) takeoff distance to 35 ft.) x 1.15

References

17.8.4.1 Schaufele, R.D., "The Elements of Aircraft Preliminary Design", Aries Publications, 2007

17.8.4.2 U.S. Code of Regulations, Title 14, Part 1 (https://www.ecfr.gov/current/title-14/chapter-I/subchapter-A/part-1)

17.8.4.3 U.S. Code of Regulations, Title 14, Part 25 (https://www.ecfr.gov/current/title-14/chapter-I/subchapter-C/part-25)

17.8.4.4 U.S. Code of Regulations, Title 14, Part 23 (https://www.ecfr.gov/current/title-14/chapter-I/subchapter-C/part-23)