12.6.1 Oswald Span Efficiency Method

This method for calculating drag due to lift is usually applied to subsonic airplanes, although Raymer shows a correction for supersonic flight as Eq. (12.51). In this section, the drag due to lift factor, K, is defined as:

$$K = \frac{1}{\pi A e} \tag{12.48}$$

where e is the airplane efficiency factor, or Oswald efficiency factor. For a wing planform with an elliptical lift distribution, e would have a theoretical value of unity. In practice wings rarely have an elliptical lift distribution (except for notable examples such as the Supermarine Spitfire), and we must also consider changes in viscous drag due to lift. Raymer provides an estimation method for e in Eq. (12.48) and (12.49) that is in terms only of aspect ratio and sweep, but excludes the effect of the fuselage on spanwise lift distribution, and changes in viscous drag due to lift. This method also produces rather low values of e for aspect ratios below about 5, and the Shevell method, described below, gives better results for aspect ratios for typical commercial airplanes.

Raymer describes an alternative method for the estimation of K in Section 12.6.2, and this method does take account of viscous forces, but it is more commonly applied to low aspect ratio and supersonic airplanes.

For subsonic airplanes with high aspect ratio planforms, an extension of the Oswald span efficiency method may be used that takes account of such factors as wing-fuselage interference and other viscous drag effects. This method is described in Dick Shevell's book "Fundamentals of Flight" in which much of the data is derived from Douglas commercial airplanes.

Using Shevell's method, the airplane efficiency factor is

$$e = \frac{1}{(\pi A\, k) + \frac{1}{us}} \tag{12.6.1.1}$$

where k is the viscous drag due to lift factor. Sample values are quoted in Shevell's text, and a general relationship can be derived from the quoted values as

$$k = \left(0.38 + 57 \times 10^{-6}\, \Lambda^2\right) C_{D_0} \tag{12.6.1.2}$$

(where the quarter-chord sweep, Λ, is in degrees).

The variable u is the planform efficiency factor which is usually between 0.98 and 1.0, but which can be assumed to be 0.99, and s is the induced drag factor due to the effect of the fuselage on the spanwise lift distribution. This is shown graphically in Shevell, but can be approximated by

$$s = 1 - 1.556\left(\frac{d_f}{b}\right)^2 \qquad (12.6.1.3)$$

where d_f is the fuselage diameter, and b the span. These equations are mostly empirical and based on analysis of flight test data at Douglas aircraft.

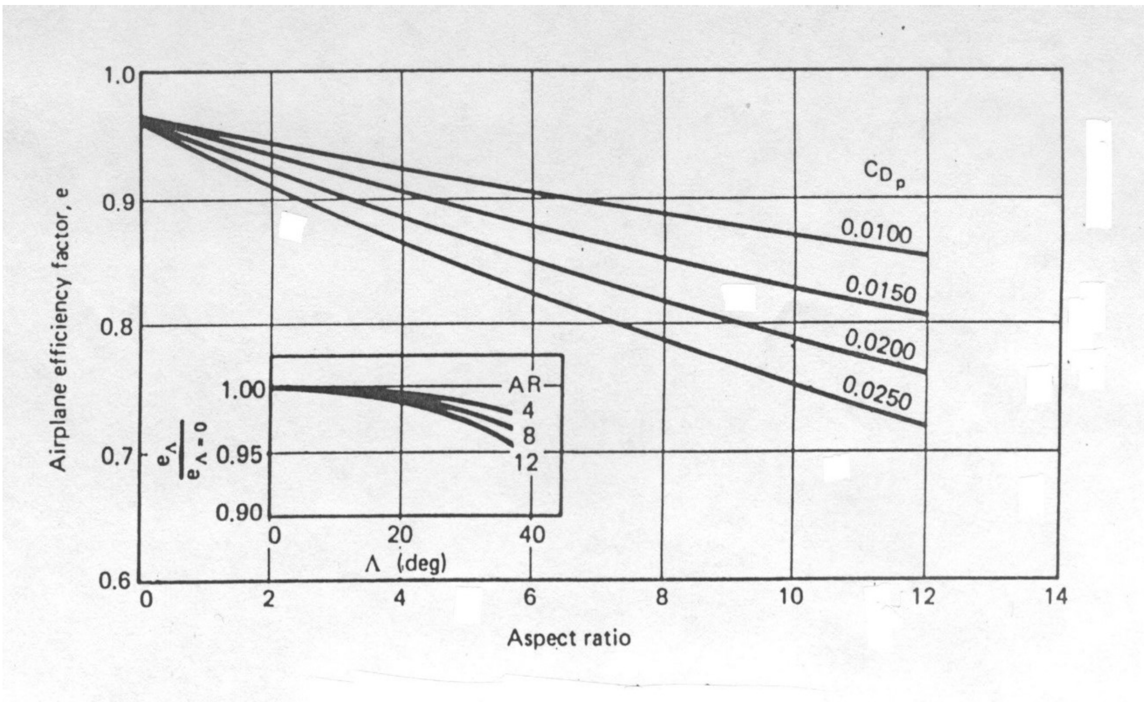


Fig 12.6.1.1 Airplane Efficiency Factor, *e*

Fig. 12.6.1.1 from Shevell is a plot of airplane efficiency factor for $u = 0.99$ and $s = 0.975$ and shows that zero-lift drag (shown in this figure as C_{D_p}) has a significant influence on e. The main body of the figure shows the value of e for $\Lambda = 0$ ($e_{\Lambda=0}$). To find the value of e for a swept wing (e_Λ), this value must be factored by $\frac{e_\Lambda}{e_{\Lambda=0}}$. Λ can be assumed to be the sweep at the quarter chord.

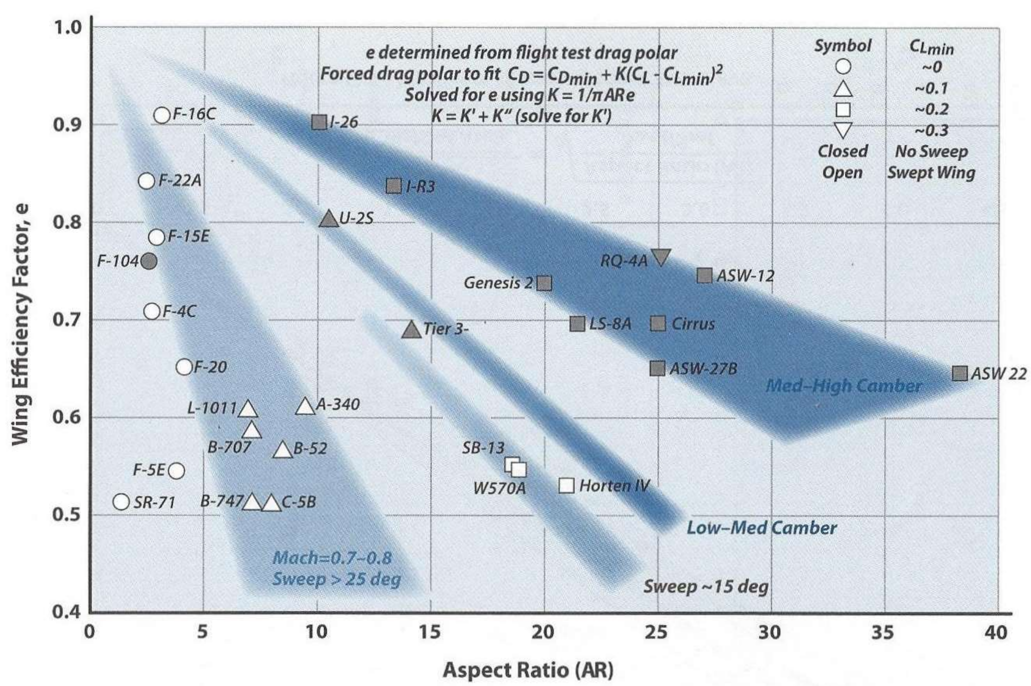
The value of e from this method is valid provided that the simplifying assumption is made that the drag polar is symmetric so that $C_{L_{min}} = 0$, even though this is not the case for most subsonic airplanes (except aerobatic aircraft). If the airfoil is non-symmetric (as illustrated in Raymer Fig. 4.9), and/or the wing is set at an angle of incidence with respect to the fuselage (see Raymer section 4.3.5), then $C_{L_{min}} \neq 0$. However, the assumption that $C_{L_{min}} = 0$ is made Raymer's book, and most other textbooks, in order to simplify subsequent performance analysis (and make life easier for students).

Compressible drag due to lift in the transonic region is described in Raymer Section 12.5.10 Transonic Parasite Drag.

If $C_{L_{min}} \neq 0$, then the values of e shown in Fig. 12.6.1.2 are not valid for any of the aerodynamics and performance equations in this textbook. Typically the value of e for an airplane is calculated by plotting C_D vs. C_L^2 using experimental data, and then drawing best fit straight lines to the data. An example is shown in an exaggerated form in Fig. 12.6.1.3. If $C_{L_{min}} \neq 0$, and the fit is made assuming that

$$C_D = C_{D_{Min}} + K\left(C_L - C_{L_{min}}\right)^2 \qquad (12.6.1.4)$$

as was done for this figure, then the value of K will be higher (and corresponding value of e will be lower) than if the data were fitted to a symmetric polar. The calculated value of e using the method described above should only be used in aerodynamics and performance calculations that assume an asymmetric polar.



Source: Nicolai & Carichner

Fig. 12.6.1.2 Oswald Efficiency Factor e

For all aerodynamics and performance calculations using methods in Raymer's book, a value of e should be used that is calculated by force-fitting the data to a symmetric polar, as was done for Fig. 12.6.1.1. This will produce a lower value of K (or higher value of e) than if an asymmetric polar were assumed.

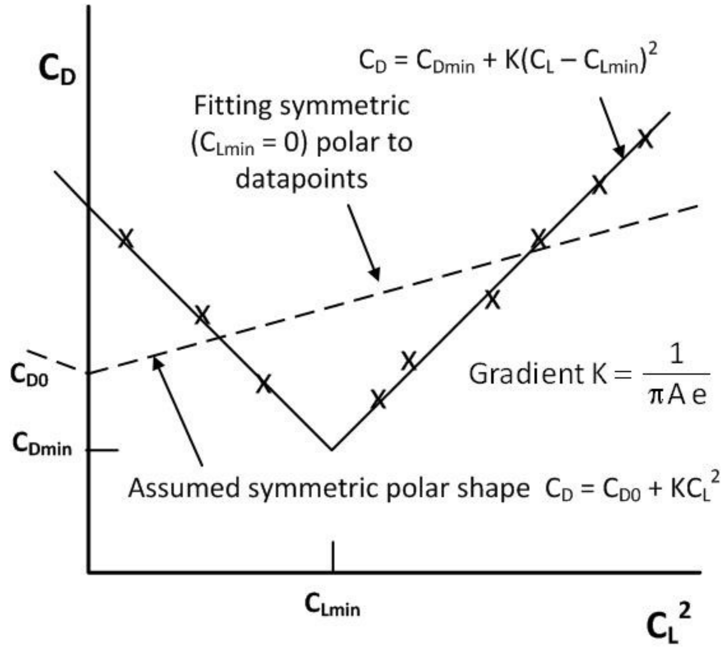


Fig. 12.6.1.3 Fitting Asymmetric and Symmetric Polars to Data

Supersonic Aircraft

For a first estimate of *e*, you may use Fig. 12.6.1.2 to select an airplane that is similar to the one that you are designing, provided that $C_{L_{min}} = 0$ for your airplane (these airplanes are marked with a circle in the figure). For wings with a sharp leading edge, quantification of *e* using the leading edge suction method is described in Raymer Section 12.6.2.