5.3 Wing Loading

Stall Speed

The approximate wing $C_{L_{max}}$ is given by Raymer in Eq. (5.7) as

$$C_{L_{\max}} \cong 0.9\left\{\left(C_{l_{\max}}\right)_{flapped}\frac{S_{flapped}}{S_{ref}}+\left(C_{l_{\max}}\right)_{unflapped}\frac{S_{unflapped}}{S_{ref}}\right\} \tag{5.7}$$

Use Appendix D (or other data on the airfoil section that you have selected) for the maximum unflapped section lift coefficient, and Table 12.2 for the lift increment for the flap system that you have selected. The difficult question to answer is what kind of flap system to use. For given takeoff and landing field length requirements a simple flap system may result in a wing that is larger than optimum for the cruise condition, resulting in increased weight and drag. A complicated flap system may permit the wing to be smaller, but will result in increased design cost, weight and maintenance.

An interesting example of optimizing the flap system for the mission is for a derivative where the wing was already designed (so the wing area was fixed), but the existing flap system was more complex than necessary for the new mission requirements. The Boeing 747-100 had a triple-slotted flap system. A derivative of the Boeing 747 was the 747SP (Special Performance), which was designed for long, thin (i.e. low traffic) routes. This derivative was purchased by Pan Am (and these airplanes were later flown by United Airlines after they took over Pan Am’s routes) and CAAC for transpacific routes such as San Francisco – Hong Kong, and San Francisco - Beijing. The reduction in the number of seats available resulted in a fuselage that was shortened by 48 ft, so that more fuel could be added for the longer range. This resulted in a lower landing weight, so that the complex flap system was no longer required. The designers decided to replace the triple-slotted flaps with single-slotted flaps, which were lighter, offered less drag (the mechanism for the triple-slotted flaps was housed in underwing fairings called canoes, and these could be removed), and lower maintenance.

Takeoff Distance

In Raymer Fig. 5.4 the curves for propeller aircraft are undoubtedly for constant speed (i.e. variable pitch) propellers. Fixed pitch propellers typically operate at off-design condition at takeoff (the selected pitch is usually optimized for climb) resulting in a much lower efficiency. These curves are therefore invalid for fixed pitch props. The units on the horizontal axis may be confusing. *T/W* and *BHP/W* are not numerically equal, but the propeller curves have been adjusted for the numerical values for *BHP/W*, and the jet values apply to the numerical values of *T/W*. If you try to calculate the equivalent thrust for a propeller-driven aircraft and then use the *T/W* scale, you will get the wrong answer.

Referring to Fig. 5.4, Raymer states "The takeoff lift coefficient is the actual lift coefficient at takeoff, not the maximum lift coefficient at takeoff conditions as used for stall calculations". The original data for this figure appears in Loftin (Ref. 1). That report makes it clear that that is true for the propeller-driven aircraft curves only. The lift coefficient to be used for jet aircraft is the maximum lift coefficient at takeoff conditions. The curves in Figure 5.4 for jet aircraft represent FAR takeoff field length and not balanced field length. As Raymer states on page 98, these two values may, or may not, be the same, depending on whether the balanced field length is greater than the all-engine takeoff to 35 feet factored by 1.15.

Climb and Glide

Eqs. (5.28) and (5.29) can be combined as

$$\frac{T}{W} = \frac{D}{W} + G = \frac{q\,C_{D_0}}{\frac{W}{S}} + \frac{W}{S}\frac{1}{q\pi Ae} + G \tag{5.3.1}$$

This is an expression for *T/W* as a continuous function of *W/S*, and may be used as a constraint line for cruise capability with a specified climb gradient potential (*G*) and dynamic pressure (*q*).

Often the cruise capability is specified at the start of cruise, with weight specified conservatively as the MTOGW, and with specified cruise speed and altitude (e.g., Mach 0.82 at 31,000 ft) and climb capability (e.g., 300 ft/min). The climb gradient is climb rate divided by forward velocity.

The wing loading at which the required *T/W* is a minimum may be found by taking the differential of Eq. (5.3.1) with respect to *W/S* and setting to zero:

$$\frac{d\left(\frac{T}{W}\right)}{d\left(\frac{W}{S}\right)} = -\frac{qC_{D_0}}{\left(\frac{W}{S}\right)^2} + \frac{1}{q\pi A\,e} \tag{5.3.2}$$

Rearranging terms:

$$\left(\frac{W}{S}\right)_{\min\frac{T}{W}} = q\sqrt{C_{D_0}\,\pi\,A\,e} \tag{5.3.3}$$

We could then put this expression for W/S back into Eq. (5.3.1) and obtain Raymer's Eq. (5.31) which is

$$\frac{T}{W} = G + 2\sqrt{\frac{C_{D_0}}{\pi A e}}$$
(5.31)

This expression is only valid for the condition that the airplane is flying at a value of *q* for which *T/W* is a minimum.

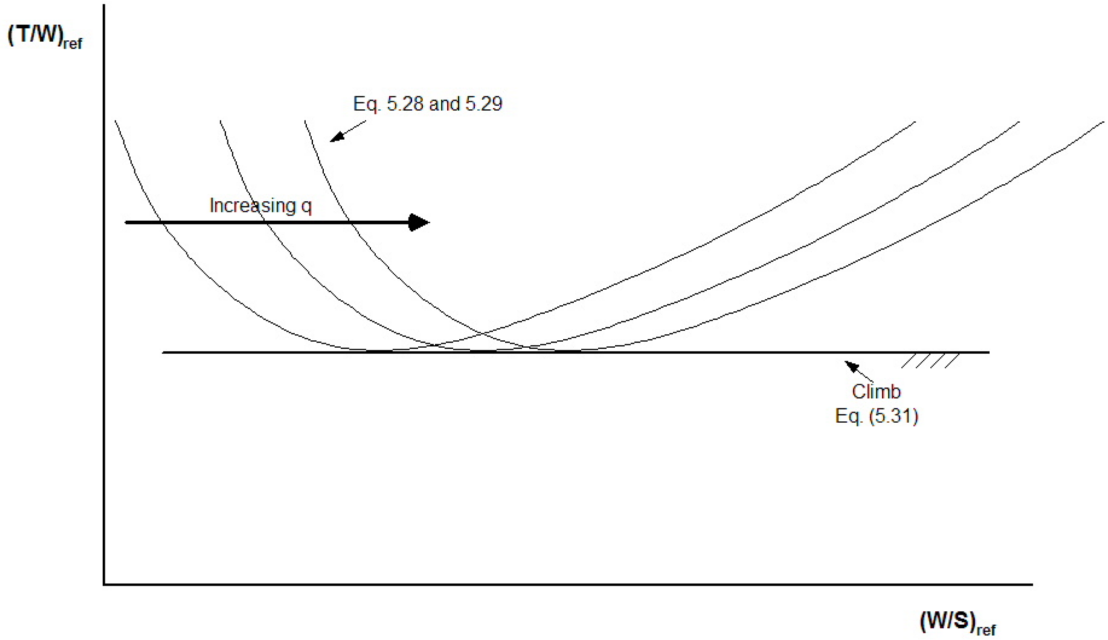

Fig. 5.3.1 Required *T/W* for climb as a function of *W/S* and *q*

For small climb angles (which is usually the case for commercial airplanes) we can assume that $L = W$, and Eq. (5.28) can simply be written as

$$\frac{T}{W} = G + \frac{1}{\left(\frac{L}{D}\right)_{climb}} \qquad (5.3.4)$$

This is true for all values of *q*, but it is only true as the minimum value of *T/W* provided that *L/D* is maximized, which can be achieved through the appropriate selection of *q*.

Notice that for the condition of $(L/D)_{max}$ then

$$\left(\frac{L}{D}\right)_{\max} = \frac{1}{2\sqrt{\frac{C_{D_o}}{\pi A e}}} \qquad (3.4.2)$$

This relationship was established in the annotations to Section 3.4.

As illustrated in Fig. 5.3.1, for maximum climb gradient the pilot could select the value of *q* (i.e., speed for a given density altitude) so that the aircraft wing loading for that particular flight is at the minimum point on the curve.

In practice, climb speeds immediately after takeoff (which are the critical design conditions) are specified by the FARs. This issue is discussed in the annotations to Section 17.3. Fortunately the specified climb speeds are a function of stall speed, which is in turn a function of wing loading for a specified wing design. Although the pilot may not be able to select the optimum *q*, the point on the generalized *T/W vs W/S* curve remains the same, so that the climb *T/W* required to satisfy the FARs is still independent of *W/S*.

References

1 Loftin, L.K., Subsonic Aircraft: Evolution and the Matching of Size to Performance, NASA Report N80-29245, Aug 1980