19.3 Improved Conceptual Sizing Methods

Climb and Acceleration

The derivation of Eq. (19.8) is similar to that of other range (Eq. 17.23), loiter (Eq. 17.30), or rocket performance (Eq. 21.31) equations.

Specific fuel energy (f_s) is defined in Chapter 17 as the change in specific energy per change in fuel weight (and noting the correction that a positive specific fuel energy is associated with a negative change in fuel weight):

$$f_s = -\frac{\mathrm{d}h_e}{\mathrm{d}W_f} = -\frac{\frac{\mathrm{d}h_e}{\mathrm{d}t}}{\frac{\mathrm{d}W_f}{\mathrm{d}t}} = \frac{P_S}{-CT} \qquad (17.94)$$

where P_s is defined as

$$P_s = V\frac{(T-D)}{W} \qquad (17.88)$$

We can drop the suffix from $\mathrm{d}W_f$ because a change in fuel weight is the same as a change in aircraft weight. From the equations above we can derive

$$\mathrm{d}h_e = -f_s\,\mathrm{d}W = \frac{P_s}{-CT}\,\mathrm{d}W = V\frac{(T-D)}{-CT}\frac{\mathrm{d}W}{W} \qquad (19.6.1)$$

Putting this in an integral format:

$$\Delta h_e = \int_{W_{i-1}}^{W_i} V\frac{(T-D)}{-CT}\frac{\mathrm{d}W}{W} = \frac{V\left(1-\frac{T}{D}\right)}{-C}\log\left(\frac{W_i}{W_{i-1}}\right) \qquad (19.6.2)$$

Rearranging this we can derive

$$\frac{W_i}{W_{i-1}} = \exp\left[\frac{-C\Delta h_e}{V\left(1-\frac{D}{T}\right)}\right] \qquad (19.8)$$

where

$$\Delta h_e = \Delta\left(h + \frac{1}{2g}V^2\right) \qquad (19.9)$$

In Eq. 19.8 we can substitute:

$$\frac{D}{T} = \frac{1}{\left(\frac{T}{W}\right)\left(\frac{L}{D}\right)} \tag{19.6.3}$$

where

$$\frac{L}{D} = \frac{\frac{W}{S}}{qC_{D_o} + \frac{K}{q}\left(\frac{W}{S}\right)^2} \tag{19.6.4}$$

by modifying Eq. (17.11). Change in aircraft weight during a climb segment can therefore be determined from the change in energy height (Δh_e), q, and the aircraft characteristics. Since these characteristics are a function of altitude, the climb must be broken up into a series of segments (at least two) to get reasonably accurate results.

The speed for optimum rate of climb is given by

$$V = \sqrt{\frac{\frac{W}{S}}{3\rho C_{d_o}}\left[\frac{T}{W} + \sqrt{\left(\frac{T}{W}\right)^2 + 12\,C_{D_o}\,K}\right]} \tag{17.43}$$

The dominant variable in this equation as the airplane climbs is the density term. However this equation can be written as

$$V = \sqrt{\frac{\rho_o}{\rho}}\sqrt{\frac{\frac{W}{S}}{3\rho_o C_{d_o}}\left[\frac{T}{W} + \sqrt{\left(\frac{T}{W}\right)^2 + 12\,C_{D_o}\,K}\right]} \tag{19.6.5}$$

or

$$V\sqrt{\frac{\rho}{\rho_o}} = \sqrt{\frac{\frac{W}{S}}{3\rho_o C_{d_o}}\left[\frac{T}{W} + \sqrt{\left(\frac{T}{W}\right)^2 + 12\,C_{D_o}\,K}\right]} \tag{19.6.6}$$

In this equation V is true airspeed (TAS). From Raymer's Appendix C, equivalent airspeed is given as

$$V_{EAS} = V_{TAS}\sqrt{\frac{\rho}{\rho_o}}$$

The optimum speed is still a function of *T/W*, which declines with altitude, and to a lesser extent with *W/S*, which also declines as fuel is burned. So if the airplane flies at a constant V_{EAS} it can maintain an approximately optimum airspeed. By climbing at a constant V_{EAS}, the airplane is also flying at a constant q, which from Eq. (19.6.4) implies flying at a constant L/D. In reality, a commercial airplane usually climbs at a constant indicated airspeed (V_{EAS} is V_{IAS} with corrections for compressibility and instrument error) from 10,000 ft until it reaches its cruise Mach number between 25,000 – 30,000 ft.