17.6 Energy-Maneuverability Methods

Energy Equations

Constraint analysis is described in Chapter 19, and typically involves finding the area in a plot of thrust/weight (*T/W*) vs wing loading (*W/S*) in which all the performance requirements are met. Examples are shown in Raymer on pages 589, 720, and 773. A configuration is usually selected that meets all the performance requirements and is at the minimum value of takeoff gross weight. Performance requirements either relate to takeoff and landing, or aerial maneuver capabilities. These requirements apply mostly to military aircraft, but some (such as a specified rate of climb at a given weight and altitude) also apply to commercial aircraft. An important equation for constraint analysis is derived in this section, so its application is described here.

Aerial maneuver capabilities typically involves the requirements for

- Specific excess power (P_s), described in Raymer Section 17.6, at specified speed, weight, and altitude
- Climb at specified speed, weight, and altitude
- Linear acceleration at specified speed, weight, and altitude
- Maximum speed in level flight at specified weight and altitude
- Sustained turn at specified speed, weight, and altitude
- Instantaneous turn at specified speed, weight and altitude.

All these requirements, except for the final one on the list above, can be plotted by starting with Eq. (17.89):

$$P_s = V\left[\frac{T}{W} - \frac{q\,C_{D_0}}{\frac{W}{S}} - n^2\,\frac{K}{q}\,\frac{W}{S}\right] \tag{17.89}$$

This can be rearranged to express *T/W* in terms of the other variables

$$\frac{T}{W} = \frac{P_s}{V} + \frac{q\,C_{D_0}}{\frac{W}{S}} + n^2\,\frac{K}{q}\,\frac{W}{S} \tag{17.6.1}$$

The first term on the right-hand side can be derived from Raymer Eq. (17.88) as

$$\frac{P_s}{V} = \frac{1}{g}\,\frac{dV}{dt} + \frac{1}{V}\,\frac{dh}{dt} \tag{17.6.2}$$

Armed with these two equations, we can define the performance constraint lines for almost any aerial maneuver.

Note that Eq. (17.6.1) is similar to Eq. (17.2.1) except for addition of the first term $\frac{P_s}{V}$, and the factor n^2 on the third term. The shape of the curve of *T/W* as a function of *W/S* (as shown in Fig. 17.2.1) is also similar. Note that if *n* is large (implying a high-*n* turn) then the gradient of the straight line in Fig. 17.2.1 is steep, which drives the minimum value of *T/W* for the overall curve (solid line) to a low value of *W/S*. This indicates that a larger wing (lower *W/S*) is required for an airplane with a high-*n* requirement.

In each case, once the *T/W* and *W/S* relationships have been derived for the specified conditions, the values of *T/W* and *W/S* must be factored so that they are expressed in terms of the reference values, which are the installed sea level static value of thrust, and maximum takeoff weight. I.e. the <u>input</u> value of *W/S* is

$$\left(\frac{W}{S}\right) = \left(\frac{W}{W_{to}}\right)\left(\frac{W}{S}\right)_{ref} \tag{17.6.3}$$

and the <u>calculated</u> value of *T/W* is corrected as

$$\left(\frac{T}{W}\right)_{ref} = \left(\frac{T_{sls}}{T}\right)\left(\frac{W}{W_{to}}\right)\left(\frac{T}{W}\right) \tag{17.6.4}$$

The weight at that maneuver condition will be specified, so the ratio (W/W_{to}) can be calculated easily. The thrust at that maneuver condition must be found from an inspection of the propulsion data that is normally supplied by an engine manufacturer. If an engine has not been selected yet, a similar class of engine will have to be used, and examples are provided in Raymer Appendix E. Although these thrust levels are for an uninstalled engine, whereas the actual thrust achieved at that flight condition is that of an installed engine (which is of the order of 5% less than the uninstalled value), the ratio of (T/T_{sls}) must be considered to be constant because the designer at this point does not have enough information about the variation of propulsion installation losses as a function of speed and altitude.

On a constraint plot, such as in Fig. 19.4, it is implied that the values of *T/W* and *W/S* apply to the maximum takeoff weight and sea level static thrust, and this is understood by the conceptual designer.

For the six performance constraints summarized above:

- <u>Specific excess power (P_s), described in Raymer Section 17.6, at specified speed, weight, and altitude</u>: Input the appropriate values into Eq. 17.6.1 and plot $(T/W)_{ref}$ vs. $(W/S)_{ref}$.

- <u>Climb at specified speed, weight, and altitude</u>: If the aircraft is required to climb at a constant EAS (which corresponds to constant *L/D*) in the troposphere, its true airspeed will increase with altitude. The first term in Eq. (17.6.3) therefore becomes significant. This equation may be rewritten in the form

$$P_s = \frac{V}{g}\frac{dV}{dh}\frac{dh}{dt} + \frac{dh}{dt} \tag{17.6.5}$$

From Lan & Roskam (Section 9.5.1)

$$\frac{V}{g}\frac{dV}{dh} = 0.567\,M^2 \tag{17.6.6}$$

So that

$$P_s = \left(1 + 0.567\,M^2\right)\frac{dh}{dt} \tag{17.6.7}$$

This value of P_s can be substituted into Eq. (17.6.1) and $(T/W)_{ref}$ plotted against $(W/S)_{ref}$.

If the aircraft is required to climb at a constant Mach number in the troposphere, it will decelerate with altitude, and the correction (also from Lan & Roskam) now becomes:

$$P_s = \left(1 - 0.133\,M^2\right)\frac{dh}{dt} \tag{17.6.8}$$

- Linear acceleration at specified speed, weight, and altitude: For the calculation of required *T/W* for instantaneous acceleration at a given speed, weight and altitude, the value of *dV/dt* can be substituted into Eq. (17.6.2) and the resulting value of P_s/V substituted into Eq. (17.6.1). If the aircraft is required to accelerate over a wide range of speed (e.g., from subsonic to supersonic as in the example for a Lightweight Supercruise Fighter on page 721) then simple analysis is no longer valid. Figure 17.14 shows that P_s varies significantly as an airplane accelerates, so its acceleration will not be constant. More advanced energy-maneuverability methods discussed in Section 17.6, or other trajectory optimization methods, must be used.

- Maximum speed in level flight at specified weight and altitude: If a speed is specified for a given weight and altitude, then P_s is set to zero and n is set to 1. *T/W* can then simply be plotted against *W/S* for a given value of *q*.

- Sustained turn at specified speed, weight, and altitude: Eq. (17.52) can be rearranged into the form

$$n^2 = \left(\dot{\psi}\,\frac{V}{g}\right)^2 + 1 \qquad (17.6.9)$$

The value of n^2 can therefore be calculated for a given sustained turn rate, and this value substituted into Eq. (17.6.1).

- Instantaneous turn at specified speed, weight and altitude: As described in Section 17.4, maximum instantaneous turn rate is independent of *T/W*, and therefore appears as a vertical line on a plot of *T/W* versus *W/S*. The required wing loading at the specified condition may simply be calculated from

$$\frac{W}{S} = \left(\frac{q}{n}\right) C_{L_{\max}}$$

(17.6.10)

As before, this value of *W/S* must then be converted to the reference value using Eq. (17.6.3).

P_s *Plots*

P_s contours are important to a fighter pilot because they tell the pilot the limitations of the airplane, and if the Ps plots are also estimated (or known) for the adversary aircraft, they also tell the pilot how to exploit the weaknesses of the adversary.

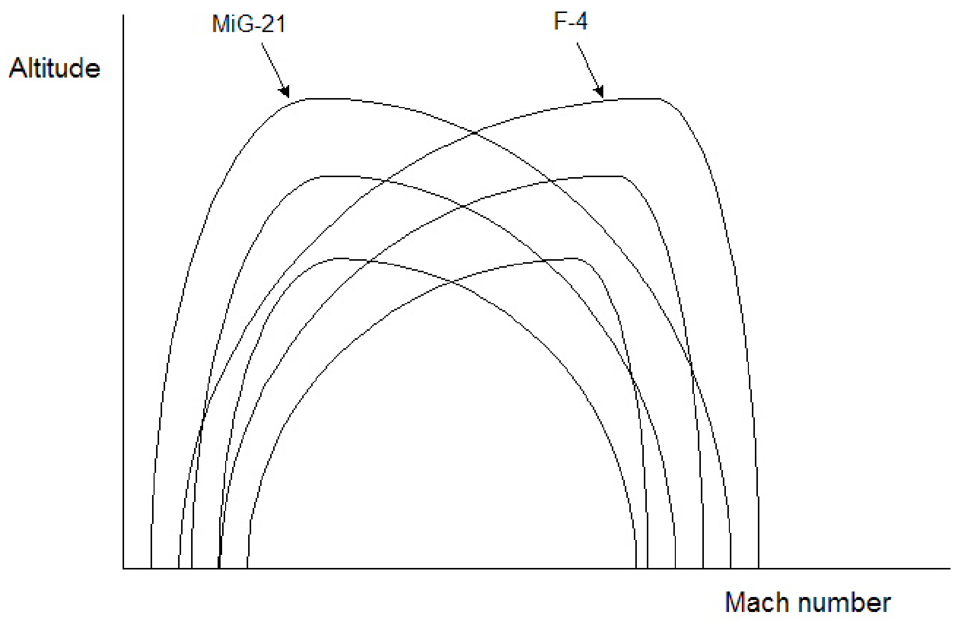

Figure 17.6.1 Notional P_s plots for two combat aircraft

For example, in Figure 17.6.1, the F-4 pilot knows that he must try to force the engagement at a Mach number where he has a P_s advantage, and not allow speeds to bleed off in combat to the extent that he has a lower P_s than his adversary.

P_s can be measured in flight test in one of three ways

- Level acceleration. The pilot accelerates the aircraft in level flight over a fairly narrow speed range, and P_s can then be calculated using the second term of Eq. (17.88). This process is repeated over a range of Mach numbers and altitudes.
- Sawtooth climb. The pilot performs a constant Mach climb, levels off to accelerate, and then climbs again at a higher Mach number, so that P_s can be calculated using the first term of Eq. (17.88).
- Constant altitude turns. Turns can be made at a value of n that for which constant speed and altitude can be maintained, so that plots can be generated in the form of Fig. 17.11.