

## 17.3 Steady Climbing and Descending Flight

### *Climb Equations of Motion*

For small climb angles, the value of  $T/W$  at a given  $L/D$  is given by Raymer Eq. 17.41 as

$$\frac{T}{W} = \frac{1}{\frac{L}{D}} + \frac{V_v}{V} \quad (17.41)$$

Where  $\frac{V_v}{V}$  is the climb gradient  $G$ .

For conditions with one engine inoperative (OEI), which will usually be the case, Eq. (17.41) must be modified to account for the loss of engine thrust so that the required  $T/W$  to meet FAR requirements, which are defined later in this section, is:

$$\frac{T}{W} = \frac{N}{N-1} \left( \frac{1}{\frac{L}{D}} + \frac{V_v}{V} \right) \quad (17.3.1)$$

where  $N$  is the number of installed engines.

The value of  $L/D$  is defined by the airplane speed (or more precisely the value of  $q$ ) and the particular airplane configuration (landing gear and flaps up or down).

The following procedure is that used by Roskam (Vol I). First, determine the clean (flaps and gear up) low-speed parabolic drag polar using Raymer Eq. (12.4) or (12.5). This polar will be used as a reference. In reality the clean drag polar would not be parabolic at a typical climb lift coefficient, because significant areas of separated flow would exist on the upper surface, with an associated increase in drag, as illustrated in Fig. 17.3.1.

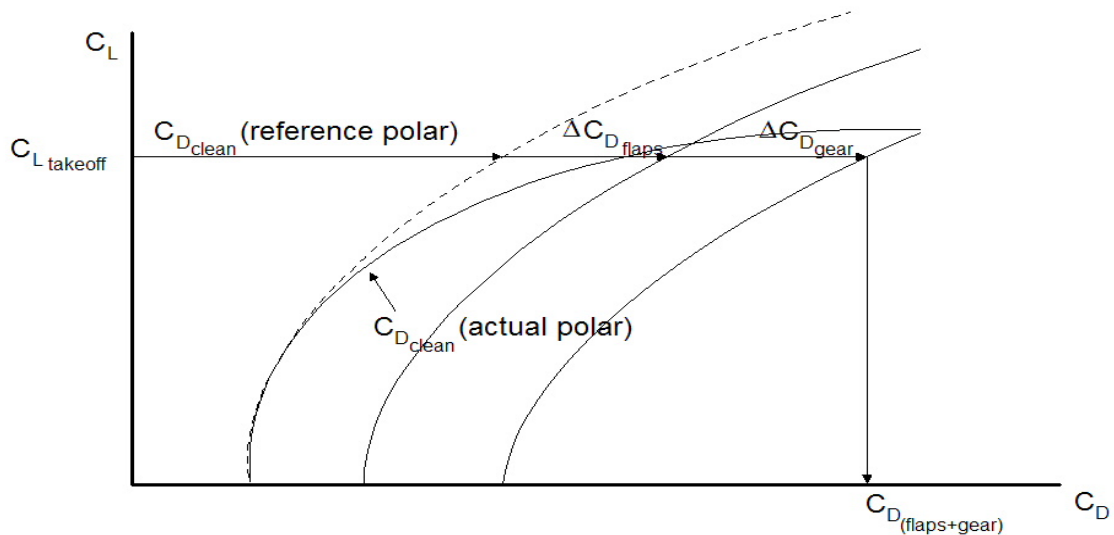


Fig. 17.3.1 Typical Drag Polars

For the aircraft configuration with takeoff flaps, the value of  $e$  must be modified to account for the change in spanwise distribution of lift (alternatively Raymer Eq. (12.62) may be used to adjust drag due to lift), and a drag increment added to take account of flap drag. Additionally, landing gear drag may also be accounted for (if the gear is down for that particular climb segment). Estimates of  $\Delta C_{D_o}$ ,  $e$  and  $C_{L_{max}}$  are given in Table 17.3.1 (taken from Roskam I, Tables 3.1 and 3.6). Since this analysis applies for the condition of one engine inoperative, there will be an additional trim drag penalty from the ailerons and rudder which is included in Eq. (17.3.4).

Configuration	$\Delta C_{D_o}$	$e$	$C_{L_{max}}$
Clean	0	0.80 – 0.85	1.2 – 1.8
Takeoff Flaps	0.0100 – 0.0200	0.75 – 0.80	1.6 – 2.2
Landing Flaps	0.0550 – 0.0750	0.70 – 0.75	1.8 – 2.8
Landing Gear	0.0150 – 0.0250	No effect	No effect

Table 17.3.1 First Estimates for  $\Delta C_{D_o}$ ,  $e$  and  $C_{L_{max}}$  for Transport Aircraft With Flaps and Gear Down

The choice of takeoff flap setting involves a tradeoff between  $L/D$  and maximum permissible  $C_L$  (under FAR rules) for that condition, and may vary between  $5^\circ$  and  $20^\circ$  for one aircraft type depending on aircraft takeoff weight, field altitude and air temperature (see Torenbeek Fig 11-11).

Climb  $C_L$  will be less than  $C_{L_{max}}$  for that condition. Since

$$C_L = \frac{L}{\frac{1}{2} \rho V^2 S} \quad (17.3.2)$$

$$\text{then } C_L = \left( \frac{V_s}{V} \right)^2 C_{L_{max}} \quad (17.3.3)$$

where  $V_s$  and  $C_{L_{max}}$  apply to the appropriate aircraft condition. For example, the second segment climb speed is defined as  $V = V_2 = 1.2 V_{STO}$  so that second segment  $C_L = C_{L_{max}}/1.44$ . From a knowledge of the  $C_L$  we can then calculate the total  $C_D$  and hence  $L/D$  for that condition. Jenkinson suggests that the trim drag ( $\Delta C_{D_{o trim}}$ ) due to the failed engine, and rudder and aileron deflection required to counteract the asymmetric flight geometry, be assumed to be 5% of the basic profile drag. For example, for the second segment climb condition in which the landing gear is up:

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D_o} + \frac{C_L^2}{\pi A e} + \Delta C_{D_{o flaps}} + \Delta C_{D_{o trim}}} \quad (17.3.4)$$

This value is inserted in Eq. (17.3.1) to obtain the required  $T/W$ .

Note that at the lift coefficient for takeoff or climb, the drag (and hence  $L/D$ ) is about the same whether the flaps are deployed for takeoff or not. (Examples of this may be found in Roskam Part VI Fig 5.5 for an unswept wing, or Torenbeek Fig 11-4 for a swept wing configuration). One might question why flaps are deployed for takeoff at all. The answer in part is that because the maximum lift coefficient is higher for the configuration with flaps deployed for takeoff, the aircraft can take off at a higher lift coefficient (the ratio of takeoff lift coefficient to maximum lift coefficient remains constant), and thus at a lower speed. An additional reason is that for transport aircraft the flaps have some Fowler action (horizontal extension of the flaps) as the flaps are extended to the takeoff position, and this gives extra lift.

For landing flap setting the pilot will normally select the maximum flap setting available. In each case, as a first approximation the values of  $C_{L_{max}}$  may be selected from Table 17.3.1. If the geometry and flap system is better defined, the value of  $C_{L_{max}}$  may be determined from Raymer Eq. (12.15) (for clean  $C_{L_{max}}$ ) and (12.21) (for increment in  $C_{L_{max}}$  due to flaps) using the value of  $\Delta C_{L_{max}}$  in Table 12.2, where

$$C_{L_{max}} = (C_{L_{max}})_{clean} + (\Delta C_{L_{max}})_{flaps} \quad (17.3.5)$$

For commercial aircraft, the most critical phases of a flight are those immediately following the failure of an engine during takeoff. FARs therefore have explicit requirements for climb capability during each phase. Additional climb requirements apply to an all-engine climb during a balked landing (i.e., following the pilot's decision not to proceed with a landing), and approach climb gradient with the critical engine inoperative.

The climb angles that must be achieved here are not necessarily the best, because the specified speeds are lower than those for best climb angle.

The FARs break up the climb flight path after takeoff into three segments:

- Takeoff, landing gear extended (commonly called 1st segment), is defined in FAR 25.121(a), and starts at the point when the aircraft reaches  $V_{LOF}$ .
- Takeoff, landing gear retracted (commonly called 2nd segment), is defined in FAR 25.121(b), and starts at the point when the landing gear is fully retracted.
- Final takeoff (commonly called 3rd segment) is defined in FAR 25.121(c), and starts at the point when the aircraft reaches 400 ft above runway surface. It ends when the aircraft reaches 1500 ft above runway surface.

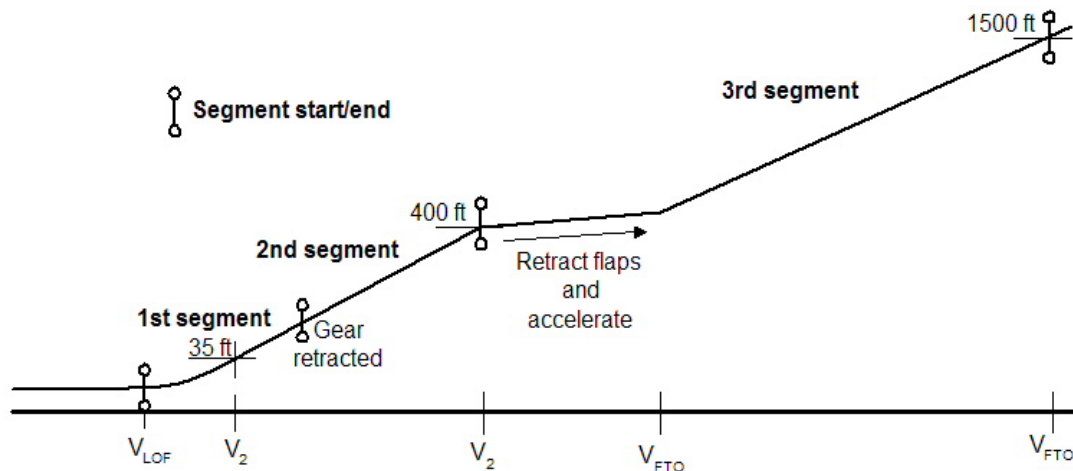


Fig. 17.3.2 FAR Climb Profile

There are also two climb requirements when the aircraft is in the approach to landing, or landing configuration:

- Approach, as defined in FAR 25.121. It should be Subsection (d), but the letter is missing in the current FARs. It will be referred to as Subsection (d) in this text.
- Landing climb, as defined in FAR 25.119.

The requirements are summarized in Table 17.3.2, which is an expanded and corrected version of Raymer Table F.4. In practice, the second segment climb nearly always sets the required climb  $T/W$ .

Operation	Operating conditions					Min. climb gradient with n engines [%]		
	Weight	Power setting	Speed [IAS]	Flaps	Landing gear	n=2	n=3	n=4
1 <sup>st</sup> segment	Takeoff <sup>a</sup>	Takeoff	$V_{LOF}$ <sup>b</sup>	Takeoff	Down	>0	0.3	0.5
2 <sup>nd</sup> segment	Takeoff <sup>a</sup>	Takeoff	$V_2$ <sup>c</sup>	Takeoff	Up	2.4	2.7	3.0
3 <sup>rd</sup> segment	Takeoff <sup>a</sup>	Max. Continuous	$V_{FTO}$ <sup>d</sup>	Up	Up	1.2	1.5	1.7
Go-around in approach config.	Max. landing	Go-around	$\leq 1.4 V_{SR}$ <sup>e</sup>	Approach <sup>f</sup>	Up	2.1	2.4	2.7
Go-around in landing config.	Max. landing	Takeoff <sup>g</sup> (AEO)	$\leq 1.23 V_{SR0}$ <sup>h</sup>	Landing	Down	3.2	3.2	3.2

Notes:

The operating conditions shown above are in some cases a simplification of the FARs. For aircraft certification the requirements of the FARs must be followed exactly. All segments are with the critical engine inoperative, except for go-around in landing configuration. The aircraft is assumed to be out of ground effect. Power settings must be obtained from the engine manufacturer.

<sup>a</sup> FAR 25.121 defines the weight at a specific point in the segment. Using TOGW gives a slightly more conservative answer.

<sup>b</sup> FAR 25.107(g) does not indicate how the value is to be calculated. Raymer (p. 548) assumes that  $V_{LOF} = 1.1 V_{STO}$  where  $V_{STO}$  is stall speed in the takeoff condition.

<sup>c</sup> FAR 25.107(c) allows the speed to be selected by the applicant within defined limits. Raymer (p. 548) suggests  $V_2 = 1.2 V_{STO}$ .  $V_2$  must be reached before the aircraft reaches 35 ft above the takeoff surface.

<sup>d</sup>  $V_{FTO}$  is final takeoff speed. FAR 25.107(g) allows this to be selected by the applicant, within defined limits. Raymer suggests a value of  $V_{FTO} \geq 1.25 V_S$ , where  $V_S$  is the stall speed in the clean condition.

<sup>e</sup> FAR 25.121(d)(1)(iii).  $V_{SR}$  is a reference stall speed, in this instance for the landing approach condition.

<sup>f</sup> Roskam suggests that approach  $C_{L_{max}}$  be 86% of landing  $C_{L_{max}}$ .

<sup>g</sup> Thrust available 8 sec. after movement of thrust levers from idle to go-around setting.

<sup>h</sup> FAR 25.119 and FAR 25.125(b)(2)(i).  $V_{SR0}$  is the reference stall speed in the landing condition.

Table 17.3.2 FAR climb requirements for multi-engine turbine-powered aircraft

The procedure for finding the value of  $T/W$  for a particular condition is

1. Estimate the value of  $C_{L_{max}}$  for that flight condition using Table 17.3.1 or using the Raymer method, outlined above.
2. Find the ratio of flight speed,  $V$ , to stall speed,  $V_S$ , for that condition from Table 17.3.2.
3. Calculate  $C_L$  for that condition from  $C_L = C_{L_{max}} \left( \frac{V_S}{V} \right)^2$
4. Calculate  $L/D$  from Eq. (17.3.4).
5. Calculate  $T/W$  for that flight condition from Eq. (17.3.1), where  $\left( \frac{V_v}{V} \right)$  is the required climb gradient in Table 17.3.1.
6. Calculate the reference value of  $T/W$  from  $\left( \frac{T}{W} \right)_{ref} = \left( \frac{T}{W} \right) \left( \frac{T_{ref}}{T} \right) \left( \frac{W}{W_{ref}} \right)$ .  $W$  is either the max. takeoff weight, or max. landing weight.

### **Best Angle and Rate of Climb – Jet**

Information on climb schedules up to cruise altitude is given in the annotations for Section 19.3, Improved Conceptual Sizing Methods, in the subsection Climb and Acceleration.