17.2 Steady Level Flight

Eq. (17.11) expresses *T/W* as a function of *W/S*, *q*, and the airplane drag characteristics C_{D_o} and *K*.

$$\frac{T}{W} = \frac{1}{\left(\frac{L}{D}\right)} = q\frac{C_{D_o}}{\left(\frac{W}{S}\right)} + \left(\frac{W}{S}\right)\frac{K}{q} \qquad (17.2.1)$$

Knowledge of the relationship between *T/W* and *W/S* at different flight conditions is important when selecting these values for aircraft sizing and trade studies (see for example Figure 19.4). Remember that these values of *T/W* and *W/S* must be corrected to the reference values, as described in the annotations to Section 5.4, when used in constraint analysis.

In the first term *T/W* is inversely proportional to *W/S*, and in the second term *T/W* is directly proportional to *W/S*. If we plot this equation with *T/W* as a function of *W/S*, then the result will look something like Figure 17.2.1.

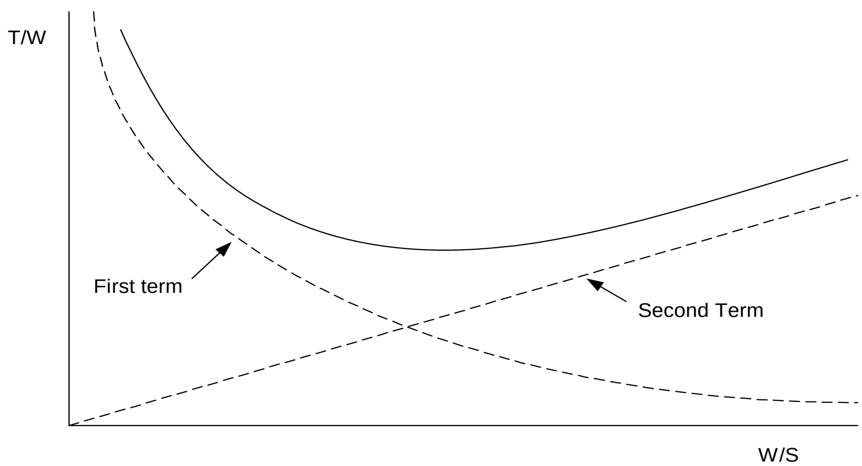

Figure 17.2.1 Cruise Thrust/Weight Ratio as a Function of Wing Loading

Notice that there is a value of *W/S* for which *T/W* is a minimum at the cruise condition. Unfortunately other design constraints, such as those for takeoff and landing, often force the wing to be larger than the optimum (or the wing loading to be lower than the optimum) for the cruise condition.

Another way of looking at Eq. (17.11) is to plot *T/W* as a function of *q* as shown in Figure 17.2.2. In this case the first term is proportional to *q* and the second term is inversely proportional to *q*. The sum of the two terms also has a minimum value of *T/W*, indicating that a value of *q* exists (and by extension a value of cruise speed *V*) for which *T/W* is a minimum. This speed is quantified in Eq. (17.13) as:

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$$V_{min\ thrust\ or\ drag} = \sqrt{\left(2\frac{W}{\rho S}\sqrt{\frac{K}{C_{D_o}}}\right)} \qquad (17.2.2)$$

If we substitute this value of $V_{min\ thrust\ or\ drag}$ into the definition of C_L, i.e.

$$C_L = \frac{W}{\frac{1}{2}\rho V^2 S} \qquad (17.2.3)$$

then we find that

$$C_{L_{min\ thrust\ or\ drag}} = \sqrt{\frac{C_{D_0}}{K}} \qquad (17.2.4)$$

By the definition of drag due to lift factor *K*, we can deduce that at this particular condition:

$$C_{D_i} = K C_L^2 = K\frac{C_{D_0}}{K} = C_{D_0} \qquad (17.2.5)$$

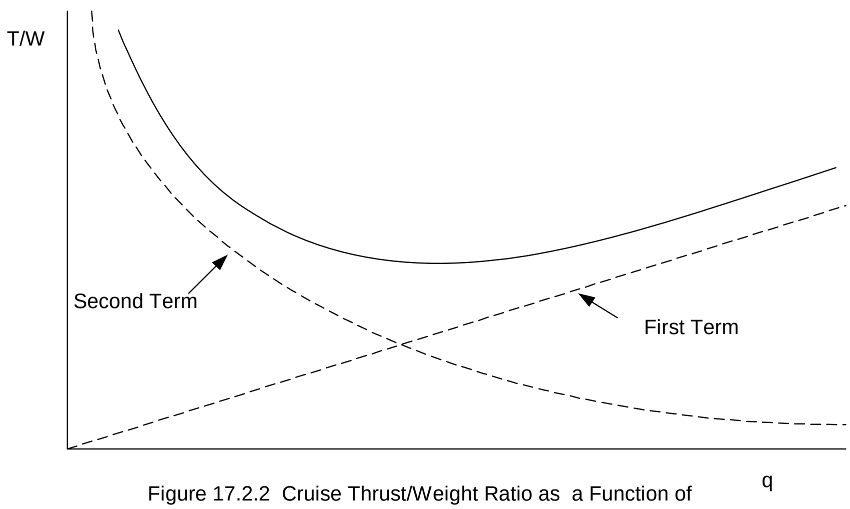

Figure 17.2.2 Cruise Thrust/Weight Ratio as a Function of Dynamic Pressure

I.e. zero-lift drag is equal to drag due to lift at the speed for minimum thrust or drag. As stated by Raymer later in this section, this speed is also the speed for maximum endurance for a jet and also maximum angle of climb for a jet.

It is also useful to know the value of *L/D* at this condition, and this can be derived from Eq. (17.14).

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$$\left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{max} = \frac{1}{C_D}\sqrt{\frac{C_{D_0}}{K}} = \frac{1}{2\,C_{D_0}}\sqrt{\frac{C_{D_0}}{K}} = \frac{1}{2\,\sqrt{C_{D_0}\,K}} \tag{17.2.6}$$

Note we have assumed that the relationship between drag due to lift and lift coefficient is parabolic and symmetrical about the C_D axis. This implies that the airfoil is uncambered, so Eq. (17.2.6) also only applies to uncambered wings. The error is small for wings with small to moderate camber, and the equation is valid for conceptual design.

For highly cambered wings we must use Raymer's Eq. (12.5) which is

$$C_D = C_{D_{\min}} + K\left(C_L - C_{L_{\min\,drag}}\right)^2 \tag{17.2.7}$$

Multiplying these terms out we get

$$C_D = C_{D_{\min}} + K\,C_L^2 - K\,C_L\,C_{L_{\min\,drag}} + K\,C_{L_{\min\,drag}}^2 \tag{17.2.8}$$

The gradient of this equation is

$$\frac{dC_D}{dC_L} = 2\,K\,C_L - K\,C_{L_{\text{min drag}}} \tag{17.2.9}$$

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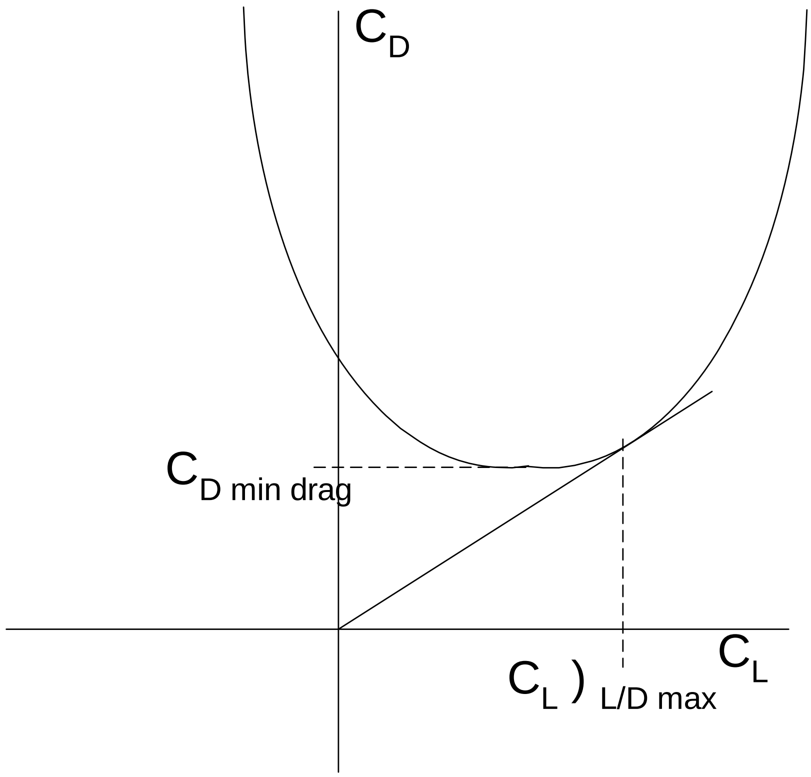

Figure 17.2.3 Finding C_L for max L/D for Cambered Airfoil

We want to find the point on the curve where the tangent from the origin touches the curve. That is where the gradient of the curve is equal to the gradient of the tangent. So we set

$$\frac{dC_D}{dC_L} = \frac{C_D}{C_L} \tag{17.2.10}$$

Or

$$C_D = C_L \frac{dC_D}{dC_L} \tag{17.2.11}$$

Substituting Eq. (17.2.10) and (17.2.11) we get

$$C_{D_0} + K C_L^2 - K C_L C_{L_{\min drag}} + K C_{L_{\min drag}}^2 = 2 K C_L^2 - K C_L C_{L_{\min drag}} \tag{17.2.12}$$

Simplifying Eq. (17.2.12) we get

$$K C_L^2 = C_{D_0} + C_{L_{\min drag}}^2 \tag{17.2.13}$$

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$$C_L\Big)_{Max\frac{L}{D}} = \sqrt{\frac{C_{D_0}}{K} + C^2_{L_{\min drag}}} \qquad (17.2.14)$$

Note that for the condition when the C_L for minimum drag is zero, Eq. (17.2.14) reduces to Raymer's Eq. (17.4).

We can then substitute the value of C_L at max *L/D* into Eq. (17.2.7) to obtain the value of maximum *L/D* for this condition. This exercise is illustrated in the accompanying spreadsheet.

Also note from Eq. (17.2.13) that

$$C_{D_0} = K\left(C_L^2 - C^2_{L_{\min drag}}\right) \qquad (17.2.15)$$

I.e., drag due to lift is equal to zero-lift drag at the condition for maximum *L/D*, as for a wing with a symmetric airfoil.

Range

Raymer mentions the step-climb procedure for commercial cruise C_L optimization in order to stay at a near-constant L/D. At altitudes up to 60,000 ft, aircraft flying over the US in an east to west direction are required to fly at even one-thousand foot intervals (e.g., 30,000 ft, 32,000 ft, 34,000 ft MSL), whereas west to east aircraft fly at odd one-thousand foot intervals. This ensures that aircraft flying in opposite directions are separated vertically by at least 1000 ft in altitude. Industrial-strength mission sizing programs will have the capability of performing the required step-climb during cruise. Typically the program will search available altitudes during cruise and find the altitude at which the aircraft will maximize specific range (nm/gal) for a given aircraft weight and Mach number. The program will then have the aircraft climb to that altitude.

For a real airplane, the flight management computer (FMC) will have much the same capability, and will recommend to the pilot the best altitude. The pilot will then request that altitude from air traffic control, and climb to it once approved. The FMC may have additional capabilities. On the Boeing 747-400, the Honeywell FMC can also adjust the Mach number so that at start of cruise on a transpacific flight the recommended Mach number might be Mach 0.86, but at end of cruise the aircraft will slow down to Mach 0.83. The pilots can also bias the results (by ranking their preference on a scale of 1 to 10) in favor of fuel economy or speed on the FMC.

Range Optimization – Jet

Raymer derives the relationship that at the speed for best range, the *L/D* is 86.6% of $(L/D)_{max}$. This is true for aircraft for which flow is incompressible and C_{D_0} and *K* are constant, as Raymer points out. For many high speed jets, including commercial

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passenger aircraft and business jets, the speed for best range using Eq. (17.25) would be above the drag divergence Mach number, and quite possibly supersonic.

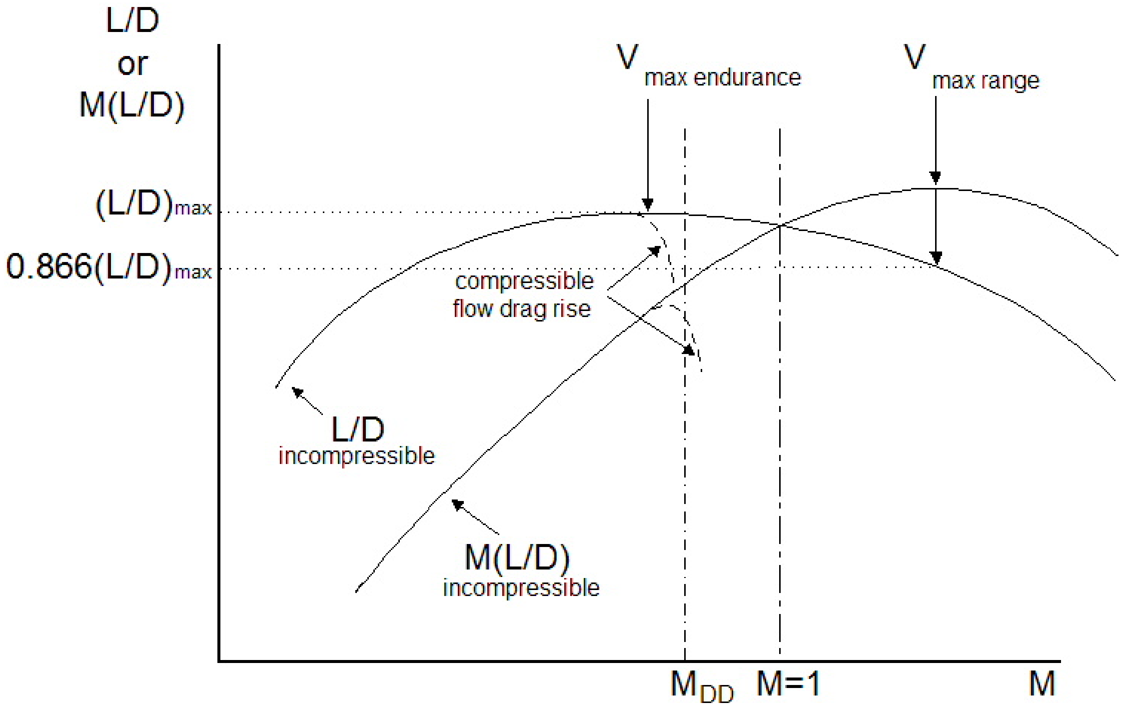

Fig. 17.2.1 Speeds for Maximum Endurance and Maximum Range for Incompressible and Compressible Flows

In Raymer Section 12.5 Parasite (Zero-Lift) Drag, in the subsection Transonic Parasite Drag, it is shown that there is a rapid rise in C_{D_0} as the drag-divergence Mach number is approached, with a resulting reduction in *L/D*. Eq. (17.25) no longer applies, but the goal of optimizing *(V(L/D))/C* does still apply. Performance engineers usually think of optimizing *M(L/D)*, which the same thing for a constant value of *C* and speed of sound. Figure 17.2.4 shows plots of *L/D* and *M(L/D)* for a configuration for which the optimum value of *M(L/D)* would be supersonic if the flow were incompressible. For compressible flow, the optimum value of *M(L/D)* is close to the drag-divergence Mach number (using the Boeing definition of drag divergence). Fortuitously, at the value of maximum *M(L/D)* in compressible flow, the value of the relationship that $L/D = 0.866\ (L/D)_{max}$ still holds roughly true, and this is good enough for initial sizing. In an industrial grade aircraft sizing program, the program will search for the value of *M* for which *(V/C)(L/D)* is optimized, taking account of the rise in C_{D_0} and increase in C_D due to lift as the drag-divergence Mach number is approached. The program may be able to take account of the cost of fuel so that the total direct operating cost is minimized. In this case, higher fuel cost would bias the optimum Mach number to a lower value.

Effect of Wind on Cruise and Loiter

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In 1975 Boeing published two important documents:

- Winds on United States Domestic Routes (Report W3410, June 1975)
- Winds on World Air Routes (Report W3412, June 1975)

These documents assembled data in tabular format for winds for individual city pairs in both the United States and worldwide at different pressure altitudes:

- Route distance: 0 – 499 nmi   Alt: 10,000/20,000/30,000/40,000 ft
- Route distance: 500 – onward   Alt: 20,000/30,000/40,000/53,000 ft

Data are provided for:

- 50% reliability winds (i.e. the wind will not be exceeded for 50% of the time) for four seasons: December-February, March-May, June-August, and September-November
- Annual wind values for 50%, 75% and 85% reliability (e.g., the wind will not be exceeded for 85% of the time on an annual basis)
- Wind standard deviations for each of the four seasons.

If a performance analyst came across a reference to "85% Boeing Annual Winds", he or she would have to find the document and look up the wind data for a given city pair. These data are now available as a computer program (Boeing PC WindTemp) that can be integrated as a subroutine into an airline's performance prediction program.

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