

## 16.3 Longitudinal Static Stability and Control

For most conceptual design studies, sizing the horizontal stabilizer based on similar horizontal tail volume coefficients, as described in Section 6.5, is sufficient. Sometimes in conceptual design and always in preliminary design the horizontal stabilizer is sized based on analysis of the configuration.

The horizontal stabilizer is sized based on at least four requirements

1. Static Margin with the c.g. at the rearmost location.
2. Control in pitch with full flaps at landing approach speed with the c.g. at the forwardmost location.
3. Takeoff rotation with the c.g. at the forwardmost location.
4. Maximum travel of the center of gravity.

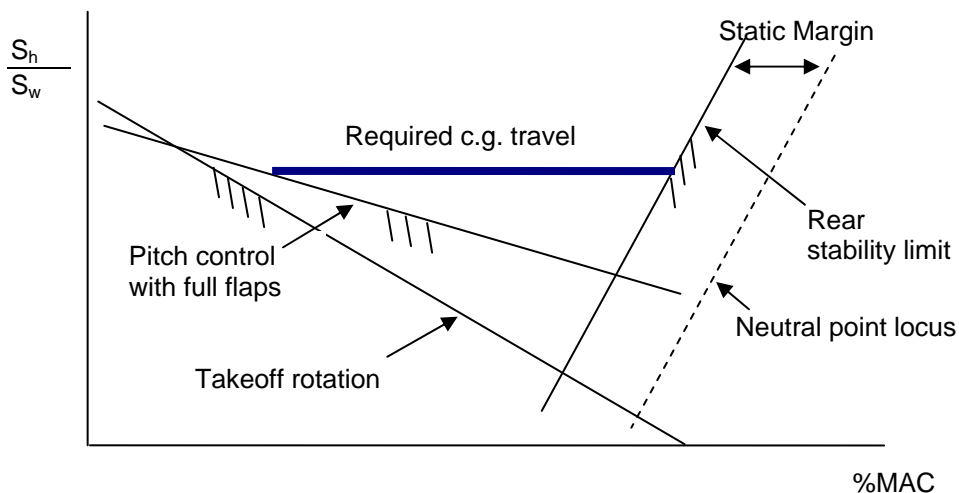


Figure 16.3.1 Notch chart c.g. travel constraints

The chart illustrated above is called a notch chart, and the objective is to minimize the horizontal tail area,  $S_h$  by matching the fore and aft c.g. travel limits to the stability and control requirements, i.e., by fitting the c.g. travel limits as low as possible into the notch. A typical value of c.g. travel for commercial airplanes is 20% MAC. This matching is done by moving the fuselage forward or aft relative to the wing. Moving the fuselage forward or aft moves the c.g. travel limits forward or aft by a lesser distance, but also affects the stability and control constraint lines. Although somewhat laborious, the analysis can easily be done using a spreadsheet such as the excel spreadsheet 'Notch' which may be found on the ADAC website under Support > Sample Spreadsheets.

The ability to calculate the constraint lines requires that the geometry of the airplane must be reasonably well defined, which is why this determination is usually performed at the preliminary rather than conceptual design stage. The method is described in detail in Torenbeek, Section 9.5.2. Note that the method described in this reference makes

allowance for the effect of the change in the weight of the horizontal stabilizer on the center of gravity. This effect is omitted in the spreadsheet referenced above and the description of the procedure described below. Torenbeek also points out that this analysis is somewhat of a simplification of the horizontal stabilizer sizing process, and a complete analysis involves other factors, such as stick forces and dynamic stability.

As Raymer states, the origin is at an arbitrary location, but for these stability and control analyses, the X origin can be conveniently located at the leading edge of the mean aerodynamic chord (LEMAC), which is also the origin of the x-axis in Fig 16.3.1.

In this analysis, only the landing approach condition is considered. The procedure for analyzing the takeoff condition is described briefly by Raymer in the subsection Takeoff Rotation on page 489.

### ***Pitching-Moment Equation and Trim***

As illustrated in Fig. 16.3.1 above, the horizontal stabilizer may be sized by the ability to trim the airplane in landing approach (with full flaps) with forward c.g. condition. This constraint line may be derived from Raymer Eq. (16.7) by expressing  $\frac{S_h}{S_w}$  in terms of  $\bar{X}_{cg}$  plus the other variables. On landing approach, engine thrust is at idle and can therefore be neglected, as can the inlet sideforce term. The critical condition is at the end of the final approach when in ground effect.

The equation is therefore of the form:

$$\frac{S_h}{S_w} = \frac{C_L (\bar{X}_{cg} - \bar{X}_{acw}) + C_{m_{w\delta f}} \delta_f + C_{m_w} + C_{m_{fus}}}{\eta_h C_{L_h} (\bar{X}_{ach} - \bar{X}_{cg})} \quad (16.3.1)$$

where

$C_L$  = wing lift coefficient, derived from knowledge of wing area, airplane weight, and approach speed

$\bar{X}_{cg}$  = non-dimensional location of the airplane center of gravity, which is a variable in this analysis

$\bar{X}_{acw}$  = non-dimensional location of the wing aerodynamic center, which is usually at 25% of the MAC

$C_{m_{w\delta f}}$  = wing pitching moment due to flaps, from Eq. (16.20)

$\delta_f$  = flap deflection in radians. A maximum of  $50^\circ$  for double- or triple-slotted flaps is suggested

$C_{m_w}$  = wing pitching moment, from Eq. (16.19)

$C_{m_{fus}}$  = fuselage pitching moment, derived from Eq. (16.25)

$\eta_h$  = ratio of dynamic pressure in tail region to free stream dynamic pressure, which (following Raymer's suggestion) can be assumed to be 0.9

$C_{L_h}$  = horizontal stabilizer lift coefficient, from Eq. (16.32)

$\bar{X}_{ach}$  = non-dimensional location of horizontal stabilizer aerodynamic center, which can be assumed to be at 25% of the horizontal stabilizer MAC.

For many conventional configurations, the wing pitching moment and fuselage pitching moment are numerically roughly equal and of opposite sign, and therefore cancel each other out. In that case they may also be neglected, but are included in this analysis.

Unfortunately, derivation of some the terms listed above is not straightforward. The wing pitching moment due to flaps, defined in Eq. (16.20), is

$$C_{m_{w\delta f}} = -\frac{\partial C_L}{\partial \delta_f} (\bar{X}_{cp} - \bar{X}_{cg}) \quad (16.20)$$

where

$\frac{\partial C_L}{\partial \delta_f}$  = change in wing lift coefficient due to flap deflection, from Eq. (16.17)

$\bar{X}_{cp}$  = non-dimensional location of center of pressure for lift increment due to flaps, which can be derived from Fig. 16.19.

Note that in Fig. 16.19,  $c'$  is the MAC of the extended chord of the wing due to Fowler action (e.g., see Fig. 12.17), so the value of  $x_{cp}$  calculated from Fig 16.9 must be non-dimensionalized by the MAC, and not by  $c'$ .

The change in wing lift coefficient due to flap deflection is given in Eq. (16.17) as

$$\frac{\partial C_L}{\partial \delta_f} = 0.9 K_f \left( \frac{\partial C_l}{\partial \delta_f} \right)_{airfoil} \frac{S_{flapped}}{S_{ref}} \cos(\Lambda_{H.L.}) \quad (16.17)$$

where

$K_f$  = empirical lift increment correction factor for plain flaps, from Fig. 16.7. For lack of better information, this will have to be used for slotted and Fowler flaps also

$\frac{\partial C_l}{\partial \delta_f}$  = theoretical wing section lift increment for plain flaps, from Fig. 16.6

$\frac{S_{flapped}}{S_{ref}}$  = ratio of wing flapped area to reference area (neglecting flap Fowler action)

$\Lambda_{H.L.}$  = flap hinge line sweep, in radians.

Figure 16.6 applies only to plain flaps, and underestimates the lift increment due to slotted flaps. Torenbeek Fig. 7-24 compares the increase in section lift coefficient for different flap configurations including a plain flap and a double-slotted Fowler flap.

From this figure it can be estimated that for double-slotted Fowler flaps, the wing section lift increment from Raymer Fig 16.6 should be factored by 1.7.

The fuselage pitching moment ( $C_{m_{fuse}}$ ) can be derived from Eq. (16.25). If the location of the wing on the fuselage is to be adjusted in order to fit the c.g. travel into the bottom of the notch, the curve may be approximated by

$$K_f = 0.00227 e^{5.1 \left( \frac{L_w}{L_f} \right)} \quad (16.3.2)$$

where

$L_w$  = distance from nose to wing root quarter chord

$L_f$  = fuselage length.

For normal fuselage arrangements, this method will suffice, but for abnormal fuselage shapes, a more detailed method, such as may be found in Ref. 16.3.1. Calculating  $C_{m_{fuse}}$  requires multiplying  $C_{m_{\alpha_{fuselage}}}$  by  $\alpha$ . This can be derived from Eq. (16.13) where  $\alpha_{OL}$  is the sum of  $\alpha_{OL_{clean}}$  and  $\Delta\alpha_{OL_{flapped}}$ . The wing incidence on the fuselage ( $i_w$ ) is a small angle (of the order of 3 degrees).

The horizontal tail lift coefficient is defined in Eq. (16.32)

$$C_{L_h} = C_{L_{\alpha_h}} \left[ (\alpha + i_w) \left( 1 - \frac{\partial \varepsilon}{\partial \alpha} \right) + (i_h - i_w) - \alpha_{OL_h} \right] \quad (16.32)$$

where

$C_{L_{\alpha_h}}$  = horizontal tail lift curve slope, from Eq. (12.6). At the end of the final approach, when in ground effect, this must be increased by about 10%, as indicated by Raymer on page 489

$i_h$  = is the horizontal stabilizer incidence, which on a commercial airplane is set by the trim wheel on the flight deck. A value of  $-10^\circ$  is suggested for the landing approach condition.

$\frac{\partial \varepsilon}{\partial \alpha}$  = change in downwash due to alpha at horizontal stabilizer, from Fig. 16.12.

When in ground effect this term must be reduced by one half, as indicated by Raymer on page 489

The zero-lift angle of attack for the horizontal stabilizer ( $\alpha_{OL_h}$ ) can be defined as

$$\alpha_{OL_h} = \alpha_{OL_h_{clean}} + \Delta\alpha_{OL_h} \quad (16.3.4)$$

where

$\alpha_{OL_{h_{clean}}}$  = is the angle of attack for zero lift with zero elevator deflection, which is usually zero

$\Delta\alpha_{OL_h}$  = is the change in zero lift alpha for the horizontal stabilizer due to elevator deflection ( $\delta_\epsilon$ ) using Eq. (16.16), and substituting  $\delta_\epsilon$  for  $\delta_f$  in Eq. (16.16) and Eq. (16.17). This is the dominant term in the equation. A maximum elevator deflection of  $-25^\circ$  is suggested.

### Static Pitch Stability

The dashed line in Fig. 16.3.1 represents the neutral point locus, or the value of  $\frac{S_h}{S_w}$  for which the airplane has neutral longitudinal static stability. The critical condition may occur at takeoff, landing approach, or cruise. This value can be calculated by rearranging Raymer Eq. (16.9) into the form:

$$\frac{S_h}{S_w} = \frac{C_{L_\alpha} (\bar{X}_{cg} - \bar{X}_{acw}) + C_{m_{\alpha_{fuse}}}}{(\bar{X}_{ach} - \bar{X}_{cg}) \eta_h C_{L_{\alpha_h}} \frac{\partial \alpha_h}{\partial \alpha}} \quad (16.3.5)$$

where

$C_{L_\alpha}$  = gradient of wing lift curve, from Eq. (12.6)

$C_{m_{\alpha_{fuse}}}$  = fuselage pitching moment derivative from Eq. (16.25). Note that the result of Eq. (16.25) is per degree, and therefore must be multiplied by  $180/\pi$

$C_{L_{\alpha_h}}$  = gradient of horizontal tail lift curve, from Eq. (12.6)

$\frac{\partial \alpha_h}{\partial \alpha}$  = downwash derivative at tail from Eq. (16.23), using  $\frac{\partial \epsilon}{\partial \alpha}$  from Fig 16.12.

From the information above, the neutral point locus can be constructed. If the airplane is to have a positive static margin (expressed as a percentage of the MAC), a line parallel to the neutral point locus can be drawn that is offset by the static margin. The foregoing analysis, along with a given allowable c.g. travel, permits Fig. 16.3.1 to be generated.

For a configuration to be stable and trimmed at a positive angle of attack, there are two requirements:

- The derivative of pitching moment with respect to angle of attack [Eq. (16.8)] must be negative.
- When the wing zero-lift angle is zero (i.e., the wing produces no lift), the airplane must exhibit a nose-up pitching moment.

The first requirement is described in Raymer Section 16.3. The second requirement is not defined in Raymer, but it is equally important.

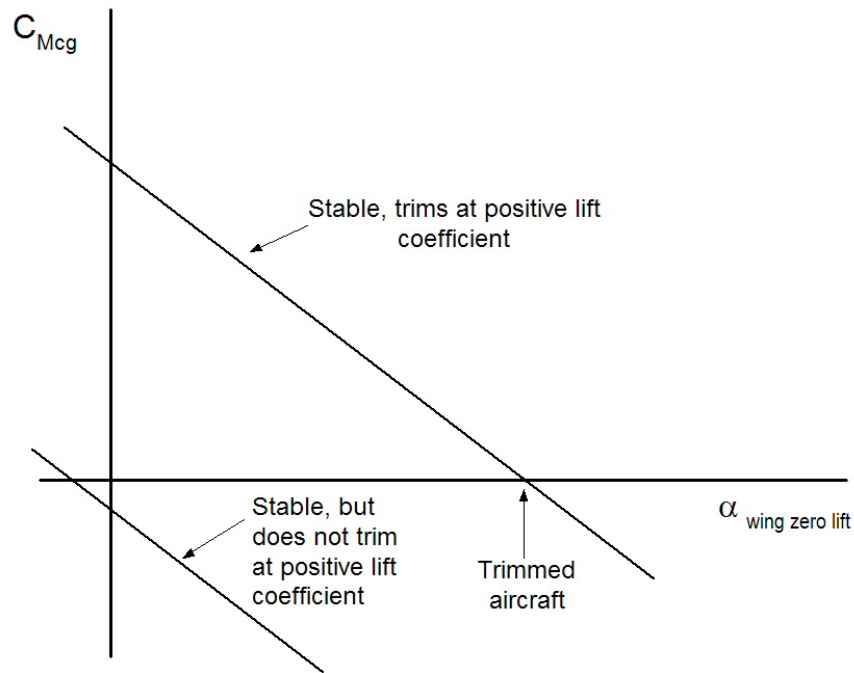


Figure 16.3.2 Feasible and Infeasible trim plots

Figure 16.3.2 illustrates two configurations. Both are stable because the derivative of pitching moment with respect to angle of attack is negative. However, only the configuration represented by the upper line will trim at an angle of attack that is positive. A configuration that is only trimmed when flying inverted would not be very comfortable for the passengers and crew. This indicates that  $C_{m_{cg}}$  (the pitching moment about the center of gravity) must be positive when  $\alpha_{\text{wing zero lift}}$  is zero. In order to achieve this, the horizontal stabilizer (if a conventional aft stabilizer is used) must provide a nose-up pitching moment (i.e. a download) at the condition at which  $\alpha_{\text{wing zero lift}}$  is zero. This does not imply that the horizontal stabilizer must provide a download at the cruise condition, although it quite often turns out to be the case for a strongly stable airplane. A canard must provide an upload at the condition at which  $\alpha_{\text{wing zero lift}}$  is zero. These conditions can be met if the aft aerodynamic surface is at a lower angle of incidence on the airplane than the forward aerodynamic surface. This arrangement is known as *décalage*, or longitudinal dihedral.

The requirement is the same for a tailless aircraft. Nose-up pitching moment can be achieved for a swept wing by applying a significant amount of washout (or nose-down twist) to the outer wing section. For an unswept trailing edge (for an unswept or delta wing), the trailing edge must be reflexed, which means that the camber line must be curved upwards at the trailing edge. For an unswept wing this results in a very inefficient wing section, so this is done very rarely. For a delta wing, with a larger MAC, the reflex does not have to be very great for adequate stability, and stable delta wings can be achieved quite easily.

On the ADAC website, under Support > Sample Spreadsheets, a spreadsheet “Cm vs alpha” is provided for testing different wing and tail configurations.

#### References

16.3.1 Perkins, C.D., and Hage, R.E., “Airplane Performance, Stability and Control”, John Wiley & Sons, 1949.