12.6 Drag due to Lift (Induced Drag)

In Raymer's book, drag due to lift and induced drag are treated as being one and the same thing (as indicated by the title of this section). As discussed in the introduction to these annotations, this approach can lead to misunderstanding by students. The situation can be clarified if induced drag (defined in these annotations as inviscid drag due to lift), and increase in profile drag due to lift (due to viscous forces), are accounted for separately. The sum of both inviscid and viscous drag due to lift is defined as "drag due to lift". This issue is discussed in detail in the annotations to Section 4.2.

It's also important to consider all the significant variables for drag and weight so that meaningful trade studies can be performed. If, for example, a trade study is performed on fuselage diameter, but the effect of fuselage diameter on induced drag is neglected, then the trade study will give the wrong results. The following procedure for calculating drag due to lift includes the effect of fuselage diameter, and other factors.

Oswald Span Efficiency Method

This method for calculating drag due to lift is usually applied to subsonic airplanes, although Raymer shows a correction for supersonic flight as Eq. (12.52). In this section, the drag due to lift factor, K, is defined as:

$$K = \frac{1}{\pi A e} \qquad (12.48)$$

where *e* is the airplane efficiency factor, or Oswald efficiency factor. For a wing planform with an elliptical lift distribution, *e* would have a theoretical value of unity. In practice wings rarely have an elliptical lift distribution (except for notable examples such as the Supermarine Spitfire), and we must also consider changes in viscous drag due to lift. Raymer provides an estimation method for *e* in Eq. (12.49) and (12.50) that is in terms only of aspect ratio and sweep, but excludes the effect of the fuselage on spanwise lift distribution, and changes in viscous drag due to lift. This method also produces rather low values of *e* for aspect ratios below about 5, and the Shevell method, described below, gives better results for aspect ratios for typical commercial airplanes.

Raymer describes an alternative method for the estimation of *K* in the next section in the book, and this method does take account of viscous forces, but it is more commonly applied to low aspect ratio and supersonic airplanes.

For subsonic airplanes with high aspect ratio planforms, an extension of the Oswald span efficiency method may be used that takes account of such factors as wing-fuselage interference and other viscous drag effects. This method is described in Dick Shevell's book "Fundamentals of Flight" in which much of the data is derived from Douglas commercial airplanes.

Using Shevell's method, the airplane efficiency factor is

$$e = \frac{1}{(\pi A\, k) + \frac{1}{us}} \qquad (12.6.1)$$

where k is the viscous drag due to lift factor. Sample values are quoted in Shevell's text, and a general relationship can be derived from the quoted values as

$$k = \left(0.38 + 57\, X\, 10^{-6}\, \Lambda^2\right) C_{D_0} \qquad (12.6.2)$$

(where the quarter-chord sweep, Λ, is in degrees).

The variable u is the planform efficiency factor which is usually between 0.98 and 1.0, but which can be assumed to be 0.99, and s is the induced drag factor due to the effect of the fuselage on the spanwise lift distribution. This is shown graphically in Shevell, but can be approximated by

$$s = 1 - 1.556\left(\frac{d_f}{b}\right)^2 \qquad (12.6.3)$$

where d_f is the fuselage diameter, and b the span. These equations are mostly empirical and based on analysis of flight test data at Douglas aircraft.

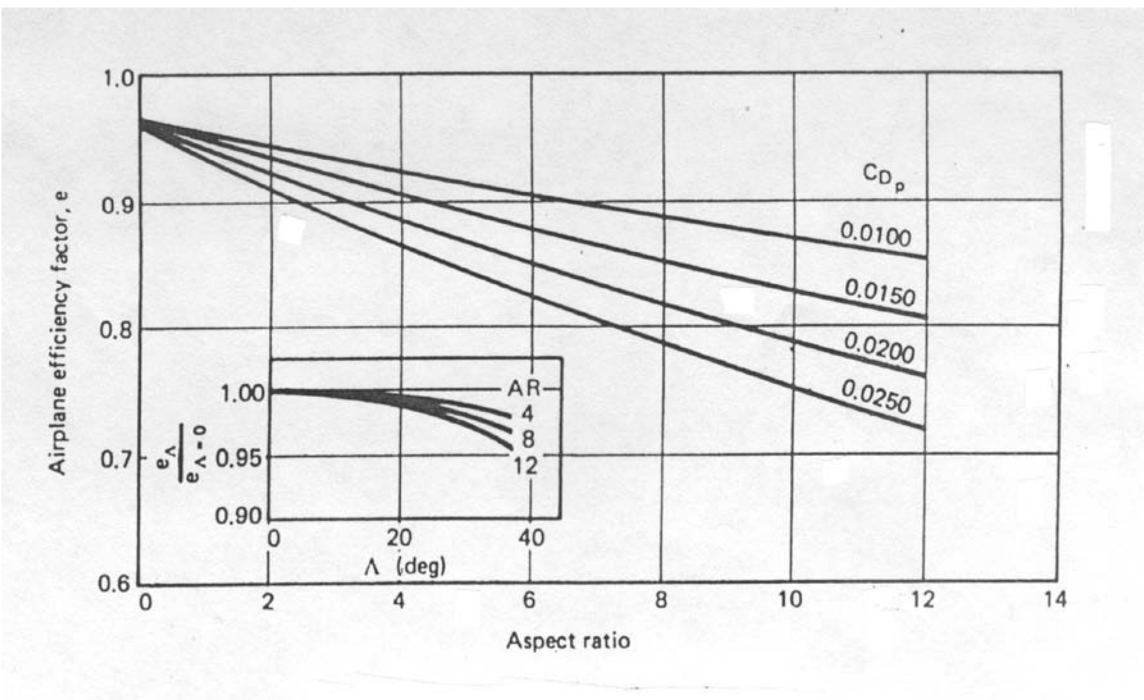

Fig 12.6.1 Airplane Efficiency Factor, *e*

Fig. 12.6.1 from Shevell is a plot of airplane efficiency factor for u = 0.99 and s = 0.975 and shows that zero-lift drag (shown in this figure as C_{D_p}) has a significant influence on e.

Compressible drag due to lift in the transonic region is described in Section 12.5 Parasite (Zero-Lift) Drag under the subsection Transonic Parasite Drag.

Leading Edge Suction Method

Fig. 12.36 shows the limiting values of induced drag factor, K, for 100% suction and 0% suction. This figure is an example for one aspect ratio. Although not stated explicitly, it can be deduced from the figure that since $K_{100} = 1/\pi A = 0.9$, then the aspect ratio for this example is about 2.83, and the spanwise loading must be close to optimum (since $e = 1$). The value of K_{100} for delta planforms of varying aspect ratios may be found in Nicolai Fig. E.5. Above a Mach number of about 3, the value of K for wings of aspect ratio of 2 or greater reduces to that predicted by supersonic linear theory or

$$K = \frac{\sqrt{M^2 - 1}}{4} \qquad (12.6.4)$$

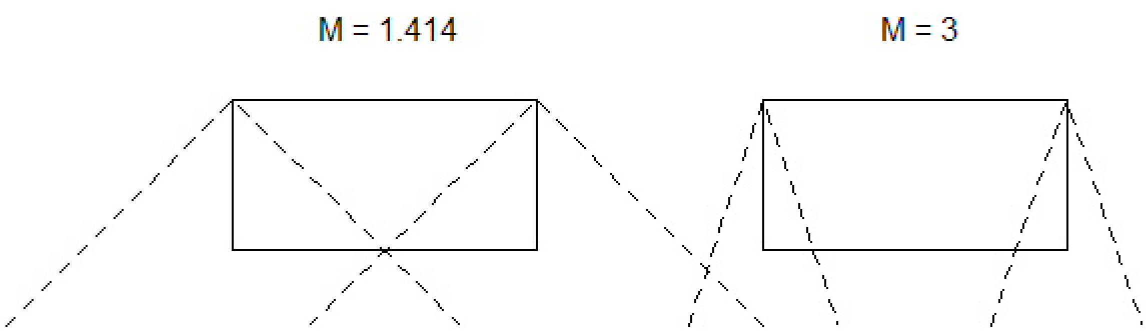

Fig 12.6.2 Cones of Influence for Aspect Ratio 2 Wing

The reduction to two dimensional flow is illustrated in Fig. 12.6.2. The Mach cone angle is given by arcsin (1/M), so for a freestream Mach number of M = 1.414 the Mach cone angle is 45^0. The influence of the wing tips exists on one half of the planform. At Mach 3 the cone angle is 19.47^0 and only 17.5% of the planform experiences any influence of the wing tips.